The New Austrian Annuity Valuation Table AVÖ 2005R

Reinhold Kainhofer[*], Martin Predota[◇], Uwe Schmock[‡]

Abstract

In this article we derive and present in detail the Austrian annuity valuation table AVÖ 2005R, which is the successor to the AVÖ 1996R table to be used for the valuation of standard annuity contracts in Austria. Its form is similar to the AVÖ 1996R in many respects: The table is a two-dimensional dynamic life table comprised of a base table for the year 2001 and age-dependent yearly trends for extrapolation into the future. The table is derived from current statistical data of the Austrian population (census of 2001, earlier censuses and yearly data since 1972), as well as from comparisons with similar German and Swiss annuity valuation tables.

In contrast to the AVÖ 1996R, the AVÖ 2005R includes security margins for model and parameter risk to account for a possible adverse development of future mortality. Thus, first- and second-order tables are available.

No explicit term for the risk of random fluctuations is included in the table, as its inclusion can only be done on a per-insurance-company level.

Keywords: Annuity valuation table, mortality reduction, AVÖ 2005R, Actuarial Association of Austria (AVÖ), Lee–Carter method, mortality projection
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1 Preface

In 1996 the previous annuity valuation table AVÖ 1996R, based on values from the census of 1991, was published. Meanwhile, the current data of the census of 2001 have been published, so it seems appropriate to update the annuity valuation table accordingly. Additionally, in the derivation of the AVÖ 1996R some optimistic assumptions have been made, most noticeable the slowing of the trends to their long-term values, which have not been observed since then.

Thus, the Austrian Association of Insurance Companies (VVÖ) commissioned the Actuarial Association of Austria (AVÖ) to develop an updated annuity valuation table, which is adapted to current mortality data and includes current assumptions and security considerations. This new table AVÖ 2005R will be presented in this article[1], together with a detailed characterization of the modifications and assumptions made in the derivation. The table is constructed as a two-dimensional dynamic mortality table with a base table for the year 2001 and age-dependent trend factors to extrapolate the mortality into the future. Although we also present a one-dimensional approximation using the method of age shifting, we strongly discourage its use for several reasons. The most important of them is that the essential prerequisites for this method are no longer properly satisfied, so the quality of this approximation is questionable. In particular, the approximated reserves for 2005 and 2006 vastly overestimate the correct values according to the exact table.

After a short introduction to the general concepts used for the AVÖ 2005R in Section 2 and a discussion of the available data in Section 3, the derivation of the table will be presented in detail in Section 4. This includes the modification of the population data to annuitants, the models for fitting the corresponding model parameters, as well as additional modifications and security margins to account for model and parameter risk. In Section 5 the table will be compared with the old table AVÖ 1996R as well as to comparable international tables like the Swiss ERM/F 1999 and the German DAV 2004-R. Finally, after several tables of resulting net (single and yearly) premiums in Section 6 and some further remarks in Section 7, the Appendix will list the concrete values of the table, including the base table, the yearly trends, the age shifts and their base tables.

2 Introduction

2.1 The AVÖ 2005R

As the number of annuity contracts has been steadily increasing to significant numbers since the publication of the previous annuity valuation table AVÖ 1996R [10], it is very important to use adequate biometric best-estimate actuarial assumptions to calculate appropriate premiums for new contracts and reserves for existing ones. The annuity valuation table AVÖ 1996R was designed for actuarial use during a period of about 10 years until new census data becomes available to adjust the table to current values. Therefore, this annuity valuation table has to be adapted and updated.

[1] The AVÖ working group that was commissioned to develop the tables presented in this article was comprised of the following persons (in alphabetical order): Martin Gaal, Reinhold Kainhofer, Franz Liebmann, Martin Predota, Adolf Schmid, Uwe Schmock, Michael Willomitzer and initially also Christina Ambach and Alois Pichler.

The result of this paper will be a dynamic life table, which should be used by Austrian insurance companies and (individual) pension funds for the calculations of premiums and provisions of annuities and similar life insurance products. It is not meant to be used as a basis for contracts that allow adjustment of premiums or payments, but rather includes considerable securities to account for the risk of adverse mortality development when the premiums are fixed. Moreover, no disablement or widow's pensions are accounted for. In these cases a different table that includes these effects needs to be used.

As calculations have shown, the net single premiums (NSP) for males calculated from the old Austrian table AVÖ 1996R are approximately 12% to 24% below the corresponding values of the German annuity valuation table DAV 2004-R, which in turn are about 4% below the values of the Swiss table.

2.2 Static and Dynamic Life Tables

A static life table gives the mortality for a fixed period, like the Austrian life table from Statistics Austria. Static life tables are principally inappropriate in life insurance for the calculation of annuities, since no mortality reduction is pictured in it.

Thus, dynamic life tables are employed for the calculation of annuities. For each age-group, the mortality is separately contained in a life table which includes the future expected change in mortality. This approach is internationally used and was already employed in the Austrian annuity valuation table AVÖ 1996R. If the development of the mortality follows the approximated trend, the dynamic life table always gives the right values. Obviously, the error made by a dynamic life table stems only from wrongly estimated trends or a wrong extrapolation model, while the error made by a static life table stems from the fact that mortality changes at all.

As one can see, using a dynamic life table, the mortality $q_x(t)$ depends not only on the age x, but also on the current year t, whereas in a static life table the influence of the generation is not included. In Austria, the life table from Statistics Austria is a static life table, hence some considerations on the mortality trend are necessary to obtain a dynamic life table.

2.3 Extrapolation of Mortality Trends

Nobody can say with certainty how the force of mortality $\mu_x(t)$ or the mortality $q_x(t)$ of a person aged x evolves with time t in the future. Based on historical data and expert judgment, one can estimate a trend for the future development of mortality, an approach which will always contain a certain level of uncertainty. Since annuities cause long-term contracts, it is important to have far-reaching projections over several decades for the mortality with sufficient statistical reliability.

To extrapolate the mortality into the future, a static base table q_x^{base} (either for a given year or for a given generation) is used as the starting point of the extrapolation. This table is typically obtained either directly from annuitant data (if available) or from official life tables by appropriate modifications to account for selection effects.

Starting from such a base table, a yearly mortality reduction factor is applied for each extrapolated year. There are several methods to determine these age- and/or birth-year dependent extrapolation factors from the available raw data, which will be shortly discussed in Section 4.5. One popular method is due to Lee and Carter [13] and will be discussed in more detail in Section 4.5.1.

As there is no data available to us about Austrian annuitants, the base table and the trend of the new AVÖ 2005R need to be obtained from population data. The base table is then multiplied by age-specific selection factors f_x^{Sel}, which account for the different mortality of annuitants in the base year compared to the whole population. These selection effects are due to the different social structure of the group of annuitants, since typically people with a higher income are more likely to sign an annuity contract, as well as due to the individual health, since healthier people are more inclined to sign an annuity contract because they expect to profit longer and thus more from such a contract than a person of poor health.

However, not only the base table but also the trend is affected by such selection effects, as Swiss investigations of annuitant mortalities and German investigations of social insurance data indicate. In other words, the selection effects between the population and the group of annuitants even increase.

The trend of the AVÖ 1996R included some optimistic predictions that could not be observed, like the linear slowing of the short-term trend to its long-term values. As a consequence, the net premiums of annuity contracts calculated with the AVÖ 1996R seem to be too low when compared with other countries. One of the aims of this table is to remove predictions that are too optimistic and use sensible predictions instead that grant a certain amount of security on the insurer's side.

Investigations of the Austrian population mortality and the Swiss annuitant mortality clearly show that the trend has not been constant, but has increased considerably in the past few decades compared to the long-term trend since 1870. To model a possible decrease of the currently high values to long-term values, or to model a further possible increase, it is of advantage to include a time-dependent modification of the trend (or equivalently a time-scaling) into the extrapolation. While the AVÖ 1996R and some other international tables include an explicit trend decrease to the long-term values, this cannot be observed in real data. To the contrary, the mortality reduction in the last decade has even further increased.

On the other hand, using a constant or even increasing trend to extrapolate into the far future will lead to vanishing mortality for all ages. While the time frame of this effect clearly lies outside the targeted time frame of this table, it is a model deficiency nonetheless. One possible solution is to add a very long-term trend decline that does not (or only to a small extent) affect mortality in the lifetime of this table, but leads to a limiting life table as the calendar year t tends to infinity. This can be done, e.g., by a time scaling

$$t \mapsto G(t) \overset{t\to\infty}{\longrightarrow} t_\infty < \infty.$$

The finite limit of the scaling function G ensures that the mortality converges in the long term towards a reasonable limiting life table. The condition that this long-term trend decline does not influence the values in the near future leads to the expansion $G(t) \approx t - 2001$ for the first decades of the 21$^{\mathrm{st}}$ century.

As a consequence, the second-order trend function of the new Austrian annuity valuation table AVÖ 2005R depends on the age of the insured person as well as on the calendar year. However, the particular choice of the very-long-term trend slowing, described in detail in Section 4.6.2, allows to rewrite this as a time-scaling with constant trend.

Combining all effects, the simplest model for a dynamic life table with extrapolation from a given base year t_0 and a long-term trend decline determines the death probability in calendar year t of a person aged x as

$$q_x(t) = q_x(t_0)e^{-\lambda_x G(t)}, \tag{1}$$

where λ_x is the age-dependent trend function and $G(t)$ is the scaled time difference to t_0 to include the long-term convergence towards a limiting life table.

Other extrapolation methods like a birth-year dependent trend or a mixture are possible and were investigated for the DAV 2004-R [6], where the classical age-dependent trend turned out to be the most appropriate one. Moreover, other forms of trend changes could be modelled by modifying λ_x in a time-dependent manner. This, however, is unnecessary, as the assumption of a trend decline cannot be justified from the data, and a possible future trend increase can more easily be included by a simple security margin.

2.4 Approximating Net Single Premiums via Age Shifting

The general form of a dynamic life table laid out above is a two-dimensional table with the age x and the calendar year t (or equivalently the year of birth $\tau = t - x$) as parameters. Thus, each generation has its own life table and the calculations need to be implemented for each of them. Consequently, actuarial values calculated from the dynamic life table depend on the year of birth as well.

For simplicity and computational reasons, some insurance companies prefer a one-dimensional approximation instead of a double-graded dynamic life table. One popular possibility to approximate double-graded dynamic life tables is the method of age shifting introduced by Rueff [16]. Instead of calculating a separate life table for each generation, one reference life table for a given generation with birth year τ_0 is selected. Each person not born in this reference year τ_0 is treated as if born in this year, but the calculations are done with a modified technical age $x \mapsto x + \Delta(\tau)$. That is, the person is made older (typically $\Delta(\tau) \geq 0$ for $\tau \leq \tau_0$) or younger ($\Delta(\tau) \leq 0$ for $\tau \geq \tau_0$) for the purpose of approximating the actuarial values

$$\ddot{a}_x(\tau) \approx \ddot{a}^{\text{base}}_{x+\Delta(\tau)}(\tau_0).$$

The values of $\Delta(\tau)$, which depend only on the year of birth τ, are chosen such that the most important actuarial values are reproduced as good as possible in a certain time period. Typically, this means that the net single premiums of annuity-dues to persons aged between 50 and 80 years are used to fit the age shifts. Most contracts include some kind of premium refund during the aggregation period, so that the net single premium at the time of annuitization has the largest influence on the premiums, while the mortality during the aggregation phase is only of secondary importance. Thus, the net single premiums for persons aged 50 to 80 need to be fitted best.

As this method of age shifting relative to a reference birth year τ_0 is only a one-dimensional approximation to a two-dimensional mortality surface, large deviations from the the exact values are possible and indeed observed.

Actually, the method of age shifting works best if the reference table—when age shifted by $\Delta(\tau)$—approximates closely the correct mortality of generation τ according to the exact table. This is for example the case if the logarithms $\log q_x(\tau + x)$ of the exact generation life tables are almost linear in x. Only conditional to similar requirements is it possible to approximate the two-dimensional life table to a satisfying degree.

Although we also present an approximated table using the method of age shifting for the AVÖ 2005R, it turns out that the requirements for a good quality of approximation is no longer satisfied. As a result, the table using age shifting grossly overestimates the net single premiums in the year 2005, while for later years the values drop even below the exact values. Thus, the table using age shifts overestimates the amount of required reserves when switching from the old AVÖ 1996R to the new table AVÖ 2005R. A more detailed discussion of this phenomenon can be found in Section 4.10.

In short, we can only discourage the use of the table using age shifting and recommend to use the exact table instead.

2.5 Security Margins: First- and Second-Order Tables

The previous Austrian annuity valuation tables, including the AVÖ 1996R, estimated only the actual mortality of annuitants in the form of second-order actuarial basis tables for Austrian annuitants. From an actuarial point of view, using second-order tables is problematic, as each projection into the future is based on assumptions that might or might not occur. An example of this is the trend decline to the long-term trend since 1870, which was assumed in the AVÖ 1996R. Since no such decline can be observed from current data and the trend keeps steadily increasing, values calculated with the old assumptions grossly underestimate the reserves and premiums that are actually needed. In other words, in these second-order tables there are no security margins included to account for such an adverse development.

Thus, the AVÖ 2005R will generate a second-order table for the best estimates of the actual mortality, as well as a first-order table including security margins to account for several different types of model and parameter risk (see Section 4.7). The magnitude of these margins are inspired by considerations that cover most of the possible risk factors, but there are still less security margins included than e.g. in the German table DAV 2004-R.

Contrary to the German and Swiss tables, the AVÖ 2005R does not include a term to account for random fluctuations. Instead, it is left to the individual actuary to include security margins for these effects, which are heavily influenced by the size and portfolio composition of an insurance company. A subsequent article will lay out a method to include such a term in practical calculations for insurance companies.

The security margins are mainly applied to the trends with the consequence that the security increases with time; this is desired since mortality projections over longer time periods are more error-prone than those for the next few years.

2.6 Notation

In this article, we will mostly follow the traditional notation common in actuarial science. The yearly effective interest rate will be denoted by r and $\alpha \in [0, 1]$ will be a level of security. Contrary to actuarial habit, in this article x will denote the age of a person independent of the sex, with a maximum age ω when needed. From the context it should be clear if a male or female is meant by x. Most formulas will hold similarly for males and females, but with different factors, except in the final tables in the Appendix, where x will denote the age of a male and y the age of a female. The calendar year will be denoted by t, and the year of birth will be written using a τ. The probability that a person aged x dies before it reaches an age of $x+1$ will be denoted by q_x, with an extrapolation trend λ_x. If this probability also depends on the calendar year t of observation, it will be denoted by $q_x(t)$. The n-year survival probability of a person aged x in calendar year t will be denoted by

$$_np_x(t) = \prod_{k=0}^{n-1} \left(1 - q_{x+k}(t+k)\right) .$$

The random variable $T_x(t)$ stands for the future lifetime of a person aged x in calendar year t, so the age at the moment of death is $x + T_x(t)$.

Whenever it is necessary to distinguish first- and second-order mortality estimates, this is indicated by $q_x^{(1)}(t)$ and $q_x^{(2)}(t)$, similarly for the trends $\lambda_x^{(1)}$ and $\lambda_x^{(2)}$.

The base tables for the exact table are denoted by $q_x^{\text{base}}(t_0)$ with t_0 for the base year and by $q_x^{\text{AS,base}}(\tau_0)$ with τ_0 for the reference year of birth for the age shifted table. Age-specific selection factors will be denoted by f_x^{Sel}, and the time scaling is written as a function $G(t)$. In the age shifted table, the birth-year-dependent age shift is denoted by $\Delta(\tau)$.

Actuarial values like net single premiums are denoted by their usual symbols $\ddot{a}_x(t)$ for an annuity-due of 1 paid in advance, $\ddot{a}_{x:\overline{n}|}(t)$ for an n-year annuity-due, and $A_x(t)$ for a whole-life insurance. The net single premium of a pure endowment of duration n issued to a life aged x in calendar year t is denoted by $_nE_x(t) = v^n\, _np_x(t)$, and the term insurance is denoted by $A^1_{x:\overline{n}|}(t)$. All further notation will be explained whenever it appears.

3 Available Data

As it was the case for the AVÖ 1996R, there is no data available about Austrian annuitants, mainly because the total volume of annuity contracts is relatively small. This is due to the comparatively small population of Austria and the fact that private annuity contracts are becoming popular only now. Thus, no pool of annuitant data is available for the derivation of the AVÖ 2005R. Even if the data of all annuity contracts were pooled, the statistical significance would be very questionable due to the small number of contracts. Hence, it is not possible to generate the whole table (like in Switzerland) or even the base table (like in Germany) from actual annuitant data.

In lack of annuitant data, the table is thus derived from Austrian population data and adjusted to annuitants by various selection factors and other modifications. Concretely, the following data about the Austrian population (provided by Mr. Hanika of Statistics Austria) were used in the derivation of the AVÖ 2005R:

- The official **Austrian Life Tables 2000/2002** [8] of the population censuses in 2001. This table is published by Statistics Austria and available up to an age of 112 years. The mortality is graduated and corrected to exclude migration effects and other unwanted influences.

- The official **Austrian Life Tables** of all censuses since 1867. Like the table for the census of 2001, these tables are graduated and error-corrected. They contain death probabilities up to an age of 95 or 100 years.

- **Yearly adjusted Austrian life tables** of Statistics Austria. These tables are raw data of the yearly mortalities for the ages from 0 to 95 years and are available since 1947. These tables are used to determine the mortality trend over 30 years, starting from 1972.

- In contrast to the derivation of the annuity valuation table AVÖ 1996R, **no data from the social insurance institution** were available.

  On the other hand, the compulsory social security has completely different characteristics than a voluntary annuity insurance, so it is questionable if data of the public social security can be used for the AVÖ 2005R.

  These data might be used to determine only selection factors due to social status. However, they would not cover selection due to personal factors like good health.

Due to the lack of Austrian annuitants data, the magnitude of the selection factors of the table are obtained mostly by comparisons with the old table AVÖ 1996R as well as the German DAV 2004-R and the Swiss ERM/F 1999. This can be justified by the strong affinity of the annuity business in Austria and Germany.

As some aspects and factors of the new table are adapted from the ones in Germany and Switzerland, it is also necessary to look at the data bases of these tables.

In contrast to the poor situation in Austria, the base table of the German DAV 2004-R was constructed using actual annuitant data of the years from 1995 to 2002. For ages above 60 years, the base table could be directly created from this data pool. For lower ages, only the selection could be directly estimated by comparing the mortality of subgroups of the insured, while the base table is still derived from the population mortality with the selection factors applied.

Similarly, the mortality trends could not be determined from the annuitant data but only from the population mortality using the official German life tables. Data from the social security insurance were then used to adjust the trends to annuitants.

In Germany the following data was used:

- Annuity contract data of 20 German insurance companies, from 1995 to 2002, pooled by the Munich Re Group and by Gen Re. This data pool consists of 1.45 Mio. years under risk and more than 33 000 deaths in the annuitization phase and 12.2 Mio. years under risk and more than 31 000 deaths in the accumulation phase. These data are a mixture of various different contracts, some of which include a money option.

- Life tables of the *Federal Statistical Office (Statistisches Bundesamt)* for West Germany from 1971/73 to 1998/2000.

- Data of the social insurance institution for West Germany from 1986 to 2002 for ages from 66 to 98.

Further details can be found in [6].

In Switzerland there is a long tradition of private pension plans, as the public pension system covers only a minimal standard of living. Hence, in Switzerland the data basis is enormous, with tables for individual contracts available since 1937 and for group contracts since 1965. Consequently, the whole Swiss annuity valuation table ERM/F 1999 (including base table and trends) could be constructed purely from annuity data. The size of the available Swiss data pool is also considerable with up to 280 000 persons under risk for males as well as females for the latest table of 1991/95. Further details can be found in [11].

4 Development of the AVÖ 2005R

4.1 The General Form of the Table

The AVÖ 2005R is implemented as a dynamic life table with a static base mortality table and extrapolation factors (trend functions) to model mortality reduction in the future. The general form of the AVÖ 2005R thus takes the official mortality tables 2000/02 for the Austrian population (as published by Statistics Austria [18]) and applies selection effects to account for different mortality effects for the group of annuitants compared to the whole population.

To determine the future mortality, this base table is extrapolated into the future by an age-dependent trend parameter λ_x, which is obtained from the population data as outlined in Section 4.5. A slight modification $G(t)$ (instead of $t - 2001$) of the time since 2001 ensures a meaningful limiting life table for $t \to \infty$. Combined, the table takes the mathematical form

$$q_x(t) = \underbrace{f_x^{\text{Sel}} \cdot \widetilde{q}_x(2001)}_{= q_x^{\text{base}}(2001)} e^{-G(t)\lambda_x} \left[\cdot f_x^{\text{RF}}\right], \quad t \geq 2001, \tag{2}$$

with the notation given in Table 1.

The AVÖ 2005R aims to be similar in form to the AVÖ 1996R, mainly because this ensures a simple upgrade path for companies already using the AVÖ 1996R table in their computer systems.

As one can see from Equation (2), it suffices to tabulate the values of $q_x^{\text{base}}(2001)$ for individual and group contracts for both males and females, as well as the λ_x for all $x = 0, 1, \ldots$ to completely determine the mortality table for any annuitant. Formally, we do not introduce a maximum age ω, but give a function to calculate probabilities for arbitrary ages. However, the survival probabilities quickly converge to 0. So in all practical calculations, using a maximum age $\omega = 120$ is recommended. Additionally, the long-term reduction function $G(t)$ needs to be specified. However, as it is meant to indicate a very long-term slowing of the trend, it is chosen in such a way that its effects are practically invisible in the next few years. Only in about 40–50 years from now one does start to see a small effect of this slowing.

In the following sections, we will discuss in detail the derivation of each of the components of Equation (2) and compare them to methods that were applied in other countries, mainly Germany or Switzerland, which are similar to Austria in their geographic location and demographic structure.

$q_x(t)$	Death probability of a person aged x in the calendar year t according to the AVÖ 2005R
$q_x^{\text{base}}(2001)$	Static base table for the year 2001, consisting of the population mortality with selection factors applied
$\widetilde{q}_x(2001)$	Official mortality table 2000/02 of the Austrian population census
f_x^{Sel}	Selection factor for a person aged x to account for different mortality levels between the general population and annuitants
λ_x	Parameter for the yearly reduction in mortality (trend), obtained from the Austrian population data of the years 1972–2002 and adjusted to annuitants, including some safety loading as detailed below
$G(t)$	Long-term decline in the trend (non-linear, yearly trend reduced to half the initial trend in $t_{1/2} = 100$ years) to ensure a limiting mortality table.
f_x^{RF}	Optional surcharge for the risk of random fluctuations; not included in the AVÖ 2005R, but left to the respective actuary

Table 1: *Notation used in the general form* (2) *of the AVÖ 2005R table.*

4.2 Austrian Population Life Table 2001

In lack of annuitant data in Austria, the AVÖ 2005R relies on the Austrian census data for the whole population and adjusts these to annuitants using selection factors. This approach was already used in the construction of the previous Austrian annuity tables (e.g. the AVÖ 1996R [10] relied on the census data of 1990/92 [9]) as well as the German DAV 2004-R.

The latest Austrian census data of the year 2001, adjusted using data from 2000 to 2002, is published in [8] by Statistics Austria [18]. We will henceforth call this table ÖSt 2000/02 in short. It contains the adjusted and graduated mortality of the Austrian population in the year 2001 by sex and age with an available age range of up to 112 years. This base table was manually extrapolated for the AVÖ 2005R to ages $x \geq 113$ using a Weibull-like function[2] (see e.g. [6, Anhang 9]):

$$q_x^{\text{base}}(2001) = 1 - \exp(-a(x+0.5)^b) \text{ for } x \geq 113 \tag{3a}$$

with parameters

$$(a,b) = \begin{cases} (2.8 \times 10^{-17}, 8.18) & \text{for males,} \\ (1.3 \times 10^{-18}, 8.79) & \text{for females.} \end{cases} \tag{3b}$$

[2]Note, however, that obviously the mortality and thus the concrete extrapolation method of the age range from 113 to 120 have practically no influence on the premiums of any relevant annuity contract.

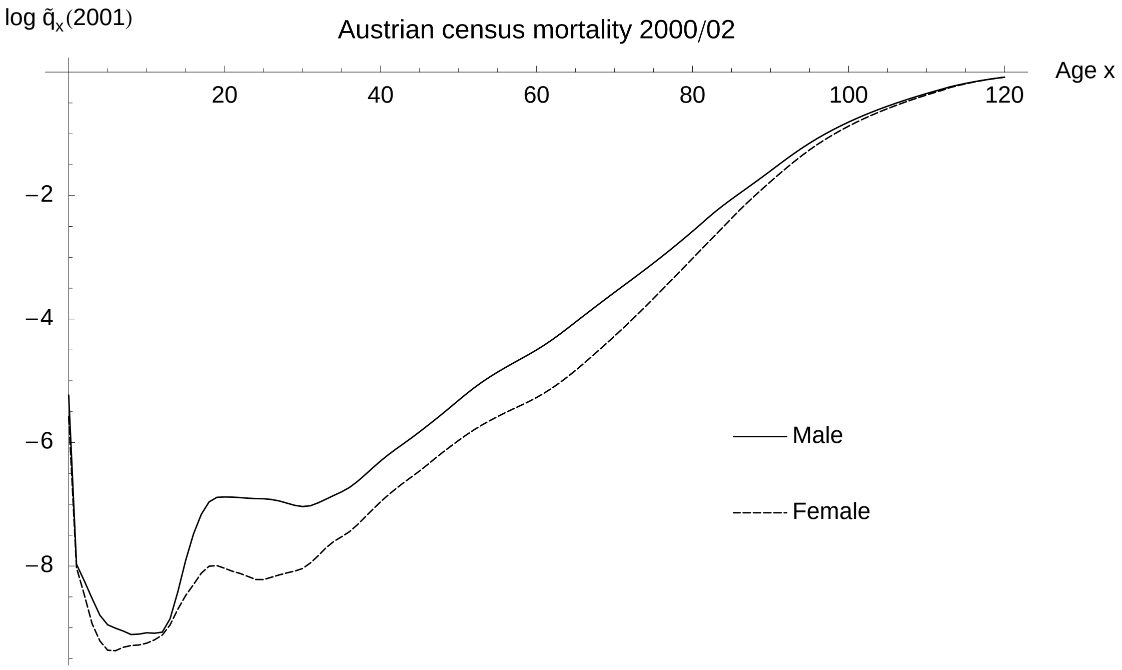


Figure 1: *Logarithm of the graduated yearly mortality of the Austrian population, data obtained by Statistics Austria through the census 2001 and adjusted using data from 2000 to 2002. The extrapolation from age 113 to age 120 and above was done manually by the authors using a Weibull-like function* (3)*.*

Although formally this extrapolation does not specify a maximum age, for all practical purposes we recommend using a cut-off age of $x = 120$ years and setting the survival probabilities to 0 above this age. In all considerations below we assume that the infinite series involving these life tables converge. In all practical calculations, the cut-off age ensures the convergence, anyway. Figure 1 shows the mortality of the Austrian population determined by the last census 2001.

4.3 Adverse Selection and Selection Factors

4.3.1 Discussion of the Selection in the Old Table

It is an undisputed fact that the subgroup of annuitants has a different mortality structure than the whole population due to selection effects. In particular, mortality among annuitants is lower, as healthier persons, who anticipate that they will benefit longer from such a policy, will be more inclined to sign an annuity contract than persons of poorer health. Furthermore, several national and international investigations (e.g. [1,6,10]) show that the social status has an enormous influence on the mortality. In particular, persons with higher wages (typically white-collar workers) have a lower mortality than persons of lower financial status (typically blue-collar workers).

For the AVÖ 1996R, these selection factors were obtained by a comparison of white- and blue-collar workers' data of the compulsory Austrian social security insurance and a comparison with the then-current German table DAV 1994-R. Since the compulsory insurance cannot capture selection effects due to individual health but only social status effects, such an approach might lead to skewed selection factors.

In Figure 2, a comparison of the selection factors of the old Austrian table AVÖ 1996R with the selection factors of the new German table DAV 2004-R[3] shows that for male annuitants the AVÖ 1996R seems to underestimate the selection effects as observed in Germany by more than 10%. For females the situation is not so dramatic, but one has to notice that the German data indicate a large difference in the selection effects of males and females.

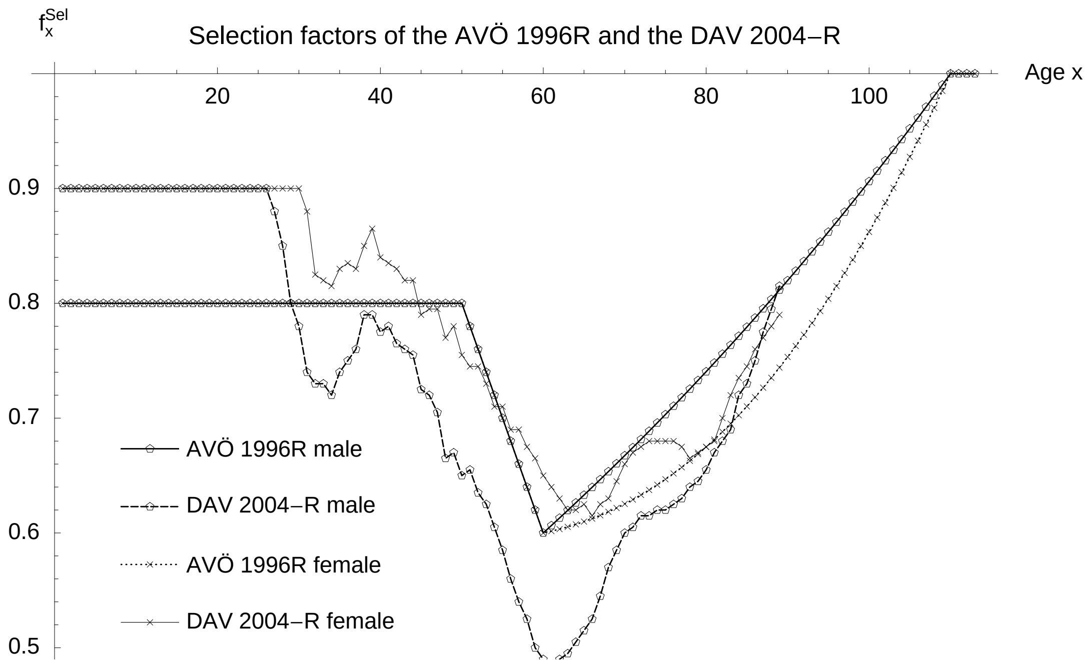


Figure 2: *In comparison to the DAV 2004-R, the male selection was grossly underestimated in the AVÖ 1996R.*

[3]The German table DAV 2004-R contains actually two different tables, a selection table and an aggregate table. The selection table takes into account the higher selection in the first five years after the start of the annuity payments. It has less over-all selection, but uses an additional factor f^1 with a value of 0.67 for males and 0.71 for females for the first year of payments. For the payment years 2 to 5, there is another factor f^{2-5} with a value of 0.876 for males and 0.798 for females to account for the higher selection in this period. The selection table cannot be used for the accumulation phase before annuitization starts, as only the mortality data after annuitization was used to determine it. The aggregate table on the other hand is averaged over the whole available data pool of annuities in the accumulation and annuitization phase and can therefore be used for the accumulation phase, too. However, it does not capture the higher selection at the beginning of the annuitization. In the derivation of the AVÖ 2005R, we use the selection factors of the German aggregate table as reference selection factors to keep the structure of the table simple.

4.3.2 Selection Factors of the AVÖ 2005R

As there are no records about Austrian annuitants, the German selection factors[4] are used as a reference for the AVÖ 2005R. These selection effects were calculated from the data pooled by Gen Re and the Munich Re Group from more than 20 German insurance companies (see Section 3).

To model the selection factors for the AVÖ 2005R, we choose a function for the selection factors that is similar to the AVÖ 1996R. The age-specific selection factor f_x^{Sel} applied to the population mortality to obtain the annuitant mortality is determined as

$$f_x^{\mathrm{Sel}} = \begin{cases} f_1 & \text{for } x \le c_1 \text{ (constant)}, \\ f_1 - (f_2 - f_1)\frac{x-c_1}{c_2-c_1} & \text{for } c_1 \le x \le c_2 \text{ (linear decline)}, \\ f_2 + (1-f_2)\frac{(x-c_2)^2}{(c_3-c_2)^2} & \text{for } c_2 \le x \le c_3 \text{ (quadratic increase to 1)}, \\ 1 & \text{for } x \ge c_3 \text{ (no selection)}. \end{cases} \tag{4}$$

	f_1	c_1	f_2	c_2	c_3
Males individual	0.8	40 years	0.51	60 years	100 years
Males group	0.8	40 years	0.61	60 years	100 years
Females individual	0.8	40 years	0.55	60 years	100 years
Females group	0.8	40 years	0.6325	60 years	100 years

Table 2: *Coefficients of the selection function in Equation* (4)*. The maximum selection for individual females is increased to a selection factor of* 0.55 *instead of* 0.6*, as a comparison with the German data suggests.*

The coefficients f_1, c_1, f_2, c_2 and c_3 are given in Table 2. As Figure 3 visualizes, this means that for ages below 40 years, the selection is kept constant at a value of 0.8, then the effect increases linearly to its maximum at age 60, which is around the typical age of retirement. Afterwards, the effect of the selection decreases until it vanishes at very old ages. Since for very old ages only the healthy population will be left and furthermore the decisions that determine the selection have passed long ago, this limit of 1 stems from actuarial considerations, but also fits very well with observations.

The form (4) has the advantages that it is mathematically easy to describe, it fits the form of the current German curves reasonably well and the same form for the selection factors was already obtained in the AVÖ 1996R by a comparison of the Austrian compulsory social security with the population mortality.

[4] In Switzerland, no modelling of selection effects of the base table is required for the generation of the ERM/F 1999, since all components of the ERM/F 1999 were obtained directly from annuity data. If one compares the base table of the ERM/F 1999 with the Swiss population of the year 1999 [20], the selection factors show a similar form with a sharp decline in the age range from 50 to 60 years, a minimum at around 55 to 65 and an increase of the factors for higher ages. While the selection factors for males increase slowly from about 0.6 for a 60-year old male to 0.7 for a 90-year old, the selection factors for females increase almost linearly from 0.57 at an age of 60 to 0.8 at an age of 90 years. Contrary to the Austrian and the German tables, the Swiss table thus has a higher selection for females than for males.

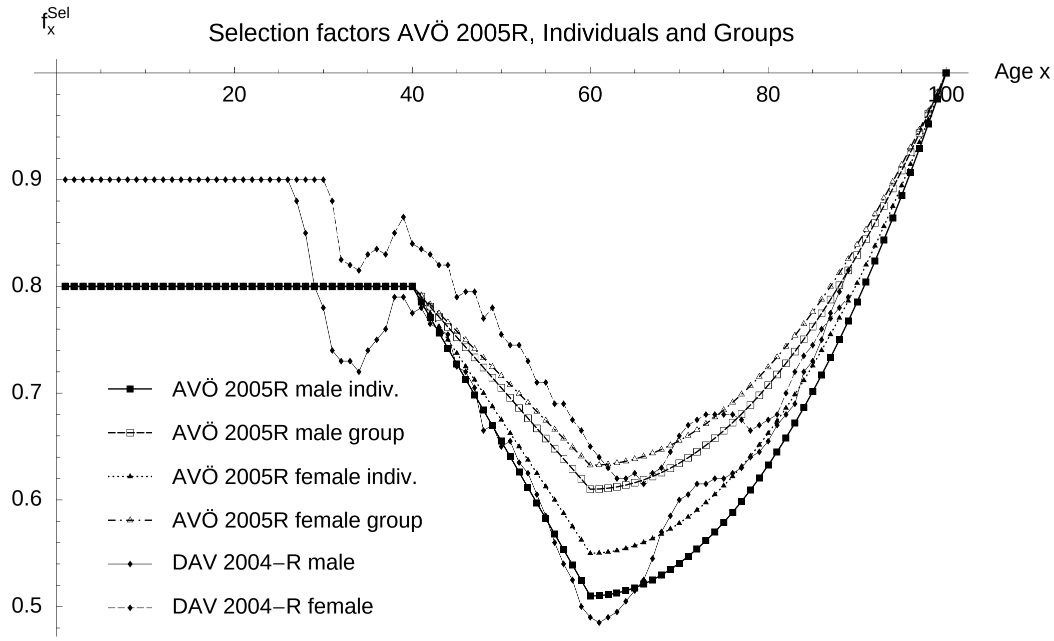


Figure 3: *Selection factors of the AVÖ 2005R, compared to the German DAV 2004-R. The selection for females was increased as discussed in Section 4.3.3. The selection factors for group contracts have a similar shape as the ones for individual contracts; only their minimum is increased.*

4.3.3 Female Selection in the Future

Figure 2 indicates that the selection has a smaller effect for females than for males in Germany. Possible reasons for this phenomenon might be:

- The female mortality is already lower than male mortality so that the selection cannot affect it as much as it can reduce male mortality.
- Couples often place annuity insurance contracts together. Since the woman—in the average Austrian marriage or partnership—is typically younger than the man, and since the selection effect is heavily influenced by the time that has passed since the contract was signed, the influence to female mortality should be lower.
- Again in the situation that a couple signs annuity contracts, according to typical role models that prevailed in the past, the decision to sign such an insurance was influenced more by the man than the woman. Consequently, the health of the man played a more fundamental role than the health of the woman.

As the latter points are already changing rapidly and it is to be expected that females will decide even more independently in the future, the selection effect for females will probably increase. For this reason, the minima of the selection factors for females were decreased by 0.05 to account for these future developments. This increase in selection for the first-order base table can thus be understood as a safety margin against adverse future developments. No such effect is included for males, so the first- and second-order male base tables coincide.

4.3.4 Selection Effects for Group Contracts

For group contracts—typically meant for corporate collective annuity insurances—the aspect of the individual health must be neglected, since the individuals cannot decide whether they want to be insured or not. Therefore only the social status influences the mortality in this case, which can be captured by a look at the data of the Austrian compulsory social insurance (which was done for the AVÖ 1996R[5]). To obtain similar levels in the AVÖ 2005R, the parameter f_2 in Equation (4) for group insurance contracts is increased by a factor 1.2 for males and 1.15 for females.

4.3.5 Other Influences on the Selection

The amount of an annuity policy has a dramatic influence on mortality[6]: Using the German annuity data, the DAV working group for the DAV 2004-R quantified this effect as a 10–15% reduction in mortality compared to the average annuitant for annuitants with a yearly annuity of more than 3 500 € and a 5–17% increase (compared to the average annuitant) in mortality for annuitants with less than 1 200 € per year. However, such an effect will not be included in the AVÖ 2005R table, which shall be understood as an average table to be applied to all annuities, independent of the insured sum. Similarly, the German table also does not include this effect.

If this increased selection for large annuities were included into the table or generally into the calculation of the premium, policy holders would conclude many smaller contracts with less selection and lower premiums, thus avoiding this penalty for large contracts.

4.4 Base Mortality Table 2001

A combination of the population mortality from Section 4.2 with the selection factors obtained in the previous Section 4.3 leads to the static base table for annuitants in the year 2001, which will subsequently be used for extrapolation. Figure 4 shows these base tables for individual annuity contracts compared to the Austrian population mortality. As one can see and expect from Table 2, the plot for the base table of group contracts does not considerably differ from the individual mortality, except in age ranges around the maximal selection. The tabulated values of the base tables for individual and group insurances can be found in Appendix A.2.

4.5 Base Trend

Since no Austrian data about annuitants is available, the trend of the mortality projection can only be obtained from the whole Austrian population and then adapted to annuitants. In Germany a similar approach was taken for the DAV 2004-R [6, p. 27ff.]. In Switzerland such a detour was not necessary and the trends could be obtained directly from the annuitant data.

To estimate the trends, it is advantageous to look at the mortality data as a matrix with components $q_x(t)$, where the age x denotes the row and the year t under consideration

[5]Since then, the rough selection level has not changed considerably.

[6]Clearly, the amount of an annuity is heavily influenced by the financial status of the insured, so these two effects are correlated.

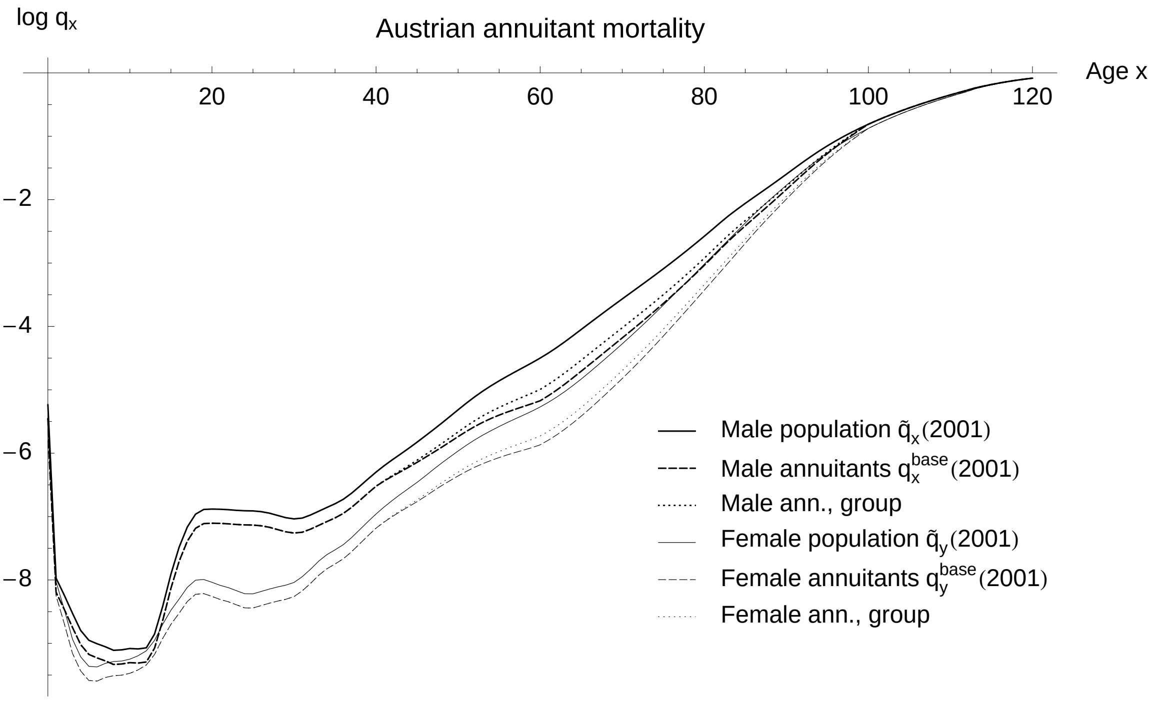


Figure 4: *Logarithm of the yearly mortality of Austrian annuitants in 2001, obtained from the population mortality with applied selection factors.*

denotes the column. In the sequel, we will approximate the natural logarithm $\log q_x(t)$ of the mortality. As a first step, the raw static life tables for each observation year are smoothed by a Whittaker–Henderson graduation to get rid of outliers and other statistical effects. This graduation is basically a discrete spline approximation that minimizes the approximation error and at the same time maximizes a smoothness measure. For details we refer e.g. to [2, Section 3.2.2.3]. Figures 5 and 6 show a plot of these graduated values for the yearly updated mortality tables since 1947, available from Statistics Austria.

The first task is to obtain approximations to these mortality surfaces, which are also well-suited for extrapolation. There are several conventional methods available that suit this purpose (see also [15] or [21]):

- The **Lee–Carter Model** [13] decomposes the logarithmic mortality surface, seen as a matrix, into[7]

$$\log q_x(t) \approx \alpha_x + \beta_x \kappa_t \,. \tag{5}$$

[7] The original Lee–Carter method decomposes the logarithm of the central death rate $m_x(t)$, which under the assumption of a constant force of mortality over each year equals the force of mortality $m_x(t) = \mu_x(t) = -\log p_x(t)$. Since we need the trends for $\log q_x(t)$, we will instead decompose the death surface $\log q_x(t)$ using the Lee–Carter methodology and apply the resulting trends to extrapolate the death probability directly from the base table obtained in the previous section. The differences between $\log q_x(t)$ and $\log(-\log p_x(t))$ are practically negligible for ages below 90 years, because $z \approx -\log(1-z)$ for $|z|$ small.

Note that here we use the assumption of a constant force of mortality throughout each year, i.e. $\mu_{x+u}(t+u) = \mu_x(t)$ for $0 \le u < 1$. In this case $m_x(t) = \mu_x(t) = -\log p_x(t)$ holds. The other common approach found in the literature is to assume linearity of ${}_u q_x(t) = u q_x(t)$ for $0 \le u < 1$ during the

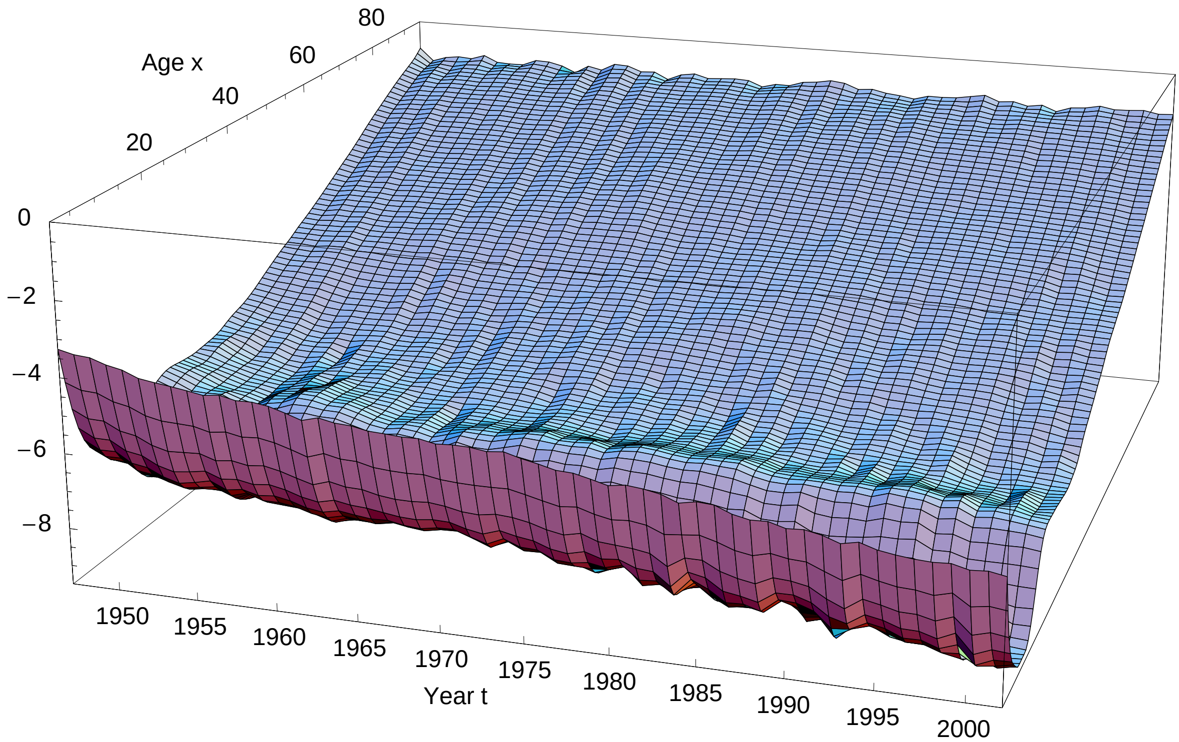


Figure 5: *Logarithm of the graduated yearly mortality of Austrian males since 1947.*

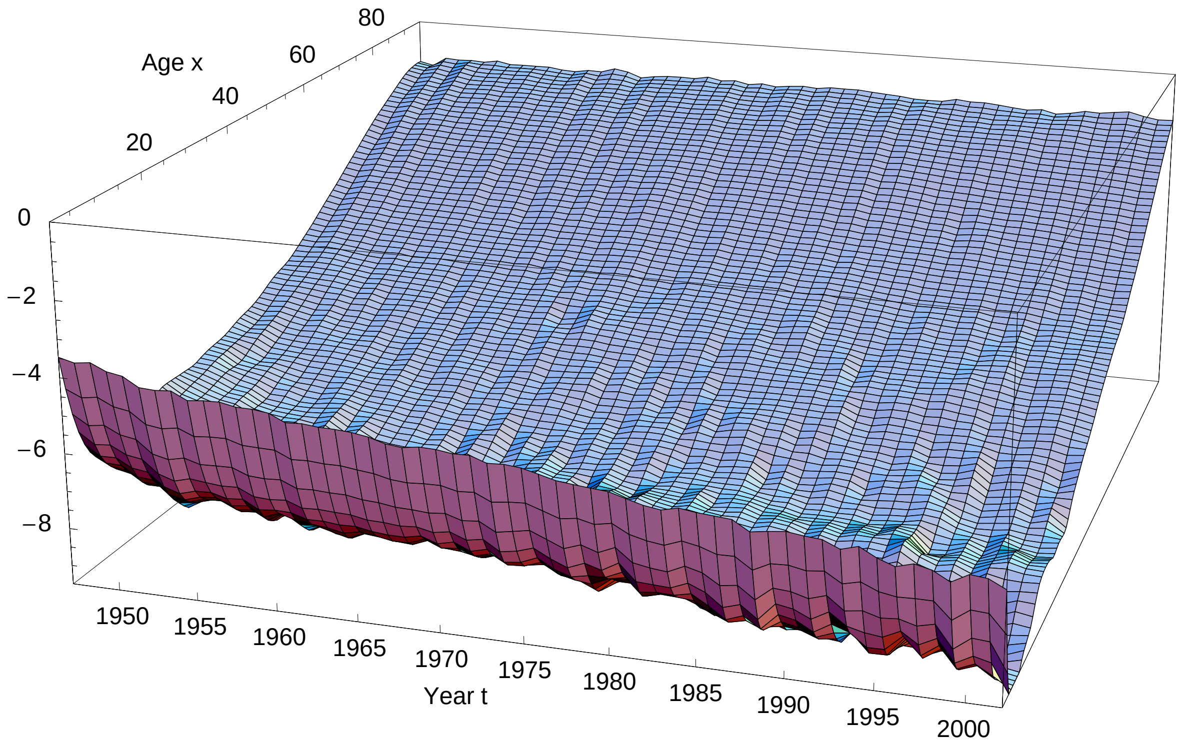


Figure 6: *Logarithm of the graduated yearly mortality of Austrian females since 1947.*

The parameters α_x, β_x and κ_t are fitted to the observed logarithmic mortality either by a simple least-squares fit, or by a Poisson regression [4,14]. The Lee–Carter Model for mortality forecasts is mainly used in the United States, but several articles are available where the model is applied to European mortality data. For example, data from the UK were considered in [14], Belgian population data in [4] and Italian data in [7]. A detailed investigation of the Austrian population mortality trends using the Lee–Carter methodology is published in [5].

A mathematical discussion of the Lee–Carter method, which was chosen for the Austrian tables, will be given in Section 4.5.1 below.

- The **Traditional Model**, the **Cohort Model** and the **Synthesis Model** investigated for the German annuity valuation tables DAV 2004-R [6] use age-specific propagation factors $F(x)$ and/or propagation factors $G(\tau)$, which are only dependent on the year of birth $\tau := t + 1 - x$ for a person aged x in year $t + 1$. The mortality reduction for year t is then approximated by

$$\frac{q_x(t+1)}{q_x(t)} = \exp(-F(x) - G(\tau)), \tag{6}$$

where the functions F and G are fitted to the observed data, with the additional restrictions $G = 0$ in the Traditional Model and $F = 0$ in the Cohort Model. The factors $F(x)$ depend only on the age and are constant for all years, i.e. the same trend is used for every year. The birth-year dependent factor $G(\tau)$ on the other hand is the same for all ages of a person born in the year $\tau = t+1-x$. The Cohort Model has the problem that it fails to correctly model reality, where the trend is also age-dependent[8].

German investigations also show that the Synthesis Model is not suitable for their mortality projections, so the Traditional Model was used in the construction of the DAV 2004-R.

- The Swiss Nolfi-Ansatz $q_x(t) = q_x(t_0)\exp(-\lambda_x \cdot (t - t_0))$ is a special case of the Traditional Model with $F(x) = \lambda_x$. In a generalized form (e.g. [11]), it models mortality as

$$q_x(t) = q_x(t_0)\,(\alpha_x + (1-\alpha_x)\exp(-\lambda_x \cdot (t-t_0)^{c_x})) \tag{7}$$

with age-specific parameters α_x, λ_x and c_x, which are fitted to the data. The Swiss annuity valuation table ERM/F 1999 employs the simple form of the Nolfi-Ansatz by choosing $\alpha_x = 0$ and $c_x = 1$ and fitting the age-dependent factors λ_x to the data.

In [6], the German working group for the DAV 2004-R also investigated the Lee–Carter Model for mortality forecasts. Although the Traditional Model was chosen for the final table, the Lee–Carter approach leads to similar projections.

calendar year t, which results in a force of mortality of the form $\mu_{x+u}(t+u) = q_x(t)/(1 - uq_x(t))$ and thus an approximation of the death probabilities using the force of mortality $\mu_{x+1/2}(t+1/2)$ by $q_x(t) = 2\mu_{x+1/2}(t+1/2)/(2 + \mu_{x+1/2}(t+1/2))$.

[8] In the last 140 years, ages above 100 years have hardly seen any mortality reduction at all, while the Cohort Model would predict the same reduction for a person aged 100 as for an 80-year old person twenty years earlier.

For the calculation of the Austrian population trends, we decided to use the Lee–Carter model, which we will discuss in more detail in the following subsection. After the trends for the whole population is determined by the singular value decomposition of the mortality matrix with components $\log q_x(t)$, the trends will be adapted to annuitants by including a constant surcharge to account for selection effects. Finally, after some cosmetic changes to ensure monotone death probabilities, the trend for old ages is increased to model prospective developments in these age ranges.

4.5.1 Lee–Carter Decomposition

The basis of the elegant Lee–Carter method [4, 12–14] is a bi-linear decomposition of the logarithmic mortality[9] $\log q_x(t)$ with $x = 0, \ldots, x_{\max}$ and $t = t_{\min}, t_{\min}+1, \ldots, t_{\min}+d = t_{\max}$ into a time-specific and an age-specific part,

$$\log q_x(t) = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t}\,, \tag{8}$$

with independent error-terms $\varepsilon_{x,t}$ satisfying $\mathbb{E}[\varepsilon_{x,t}] = 0$ and the normalization

$$\sum_{t=t_{\min}}^{t_{\max}} \kappa_t = 0 \qquad \text{and} \qquad \sum_{x=0}^{x_{\max}} \beta_x^2 = \|\beta\|_2^2 = 1\,. \tag{9}$$

Any given decomposition into α_x, β_x and κ_t with $\beta = (\beta_0, \beta_1, \ldots, \beta_{x_{\max}})^{\mathrm{t}} \neq \mathbf{0}$ can be normalized to these constraints. The first one can be achieved by the transformation

$$(\beta_x,\, \kappa_t) \mapsto \Big(\beta_x / \|\beta\|_2\,,\, \|\beta\|_2\, \kappa_t\Big)$$

and the second constraint by

$$(\kappa_t,\, \alpha_x) \mapsto \left(\kappa_t - \frac{1}{d+1}\sum_{\tau=t_{\min}}^{t_{\max}} \kappa_\tau,\; \alpha_x + \beta_x \frac{1}{d+1}\sum_{\tau=t_{\min}}^{t_{\max}} \kappa_\tau\right).$$

The trivial case $\beta_x = 0$ for all x means that there is no mortality reduction, so it will be left out in our considerations for obvious reasons: In this case no extrapolation is necessary and the α_x are already the best estimators for the future logarithmic mortality.

The normalization (9) of β_x is different from the ones found in the literature about the Lee–Carter method, where the β_x are normalized to $\sum_{x=0}^{x_{\max}} \beta_x = 1$. A reason for our choice will be given below, where β appears as normalized eigenvector.

As a consequence of the normalization of κ_t, the method of moments allows to estimate the parameter α_x as the arithmetic mean over all years,

$$\widehat{\alpha}'_x = \frac{1}{d+1}\sum_{t=t_{\min}}^{t_{\max}} \log q_x(t)\,.$$

This means that the α_x describe the mean life table over the whole time interval. The time progression, on the other hand, is completely modelled by the bi-linear term

$$(Z)_{x,t} = \log q_x(t) - \alpha_x \approx \beta_x \kappa_t\,.$$

[9] See footnote 7 on page 72.

The demonstrative interpretation of this decomposition is that the difference to the mean is decomposed into a time trend κ_t and an age-specific factor β_x that determines the strength with which the time trend affects a certain age. Mathematically, such a decomposition can be achieved by the first term of a singular value decomposition (SVD) of the $m \times n$ real matrix Z into $Z = U \cdot D \cdot V^{\mathrm{t}}$. This decomposition always exists [17], and D is the $m \times n$ diagonal matrix containing the singular values of Z in descending order. These in turn are the non-negative square roots of the eigenvalues of the matrix ZZ^{t}. The matrices U and V are orthogonal $m \times m$ and $n \times n$ square matrices containing in their columns the corresponding eigenvectors of ZZ^{t} and $Z^{\mathrm{t}}Z$, respectively.

Assuming that the random error terms $\varepsilon_{x,t}$ are i.i.d. normal random variables, the maximum-likelihood estimators for β_x and κ_t coincide with the least-squares estimators. The first term of the SVD of Z yields the ordinary least squares (OLS) approximation

$$Z \approx s\,\mathbf{u}\,\mathbf{v}^{\mathrm{t}}\,, \tag{10}$$

where s is the largest singular value and the column vectors $\mathbf{u}$ and $\mathbf{v}$ are corresponding normalized eigenvectors of ZZ^{t} and $Z^{\mathrm{t}}Z$. In several international investigations (e.g. [4,14]), this first term typically accounts for more than 90% of the variance of Z (otherwise the second term of the SVD might be used to capture effects of second order and thus an even larger part of the variance [14]).

Non-normalized ML-estimators $\widehat{\beta}'_x$ and $\widehat{\kappa}'_t$ for the coefficients β_x and κ_t can easily be obtained from the vectors $\mathbf{u} = (u_0, u_1, \dots, u_{x_{\max}})^{\mathrm{t}}$ and $\mathbf{v} = (v_{t_{\min}}, v_{t_{\min}+1}, \dots, v_{t_{\max}})^{\mathrm{t}}$ as

$$\widehat{\beta}'_x = u_x \qquad \text{and} \qquad \widehat{\kappa}'_t = s v_t\,. \tag{11}$$

A subsequent normalization of $(\widehat{\alpha}'_x, \widehat{\beta}'_x, \widehat{\kappa}'_t)$ to conditions (9) as outlined above finally yields the estimated parameters $(\widehat{\alpha}_x, \widehat{\beta}_x, \widehat{\kappa}_t)$ of the Lee–Carter decomposition.

To ensure that the SVD achieves the OLS approximation to the matrix Z, it was assumed that the $\varepsilon_{x,t}$ are i.i.d. normal random variables. In reality, this assumption does not hold, since for old ages the number of deaths is relatively small and some random fluctuations will have a larger impact on tabulated mortality of a year. Thus, the variances for these age ranges are typically larger than for younger ages. One solution proposed in [4] is a Poisson-modelling of the actual death numbers instead of the mortality. This method requires the tabulated values of lives and deaths by age and year, which are not available for the construction of the AVÖ 2005R.

Another possible approach is to normalize the matrix Z so that all ages have roughly the same variance $\widetilde{\sigma}_x^2 = 1$ via

$$\widetilde{Z}_{x,t} = \frac{\log q_x(t) - \alpha_x}{\sigma_x}\,, \tag{12}$$

where σ_x^2 is the variance of the mortality for lives aged x over the whole time horizon[10]. As the σ_x^2 are also not known exactly, one can at least find age-dependent estimators $\widehat{\sigma}_x^2$ from the residuals of a previous OLS decomposition (10) and use these estimators to normalize Z to have equal variance for all ages. The mortality is then approximated as

$$\log q_x(t) = \alpha_x + \underbrace{\widehat{\sigma}_x \widetilde{\beta}_x}_{=\beta_x} \kappa_t + \widehat{\sigma}_x \varepsilon_{x,t}\,.$$

[10]It is assumed here that the variance is only age-dependent and thus, for a given age x, is independent of the time t.

From this Lee–Carter decomposition of the mortality, one can now extrapolate to the future. The decomposition into a time-dependent trend κ_t and an age-specific factor β_x simplifies this to the extrapolation of the trend κ_t. In the simplest model, an ARIMA$(0,1,0)$ time series is employed, which models the κ_t as a random walk with drift,

$$\kappa_{t+1} = \kappa_t + \Delta\kappa + \delta_t \,.$$

The constant $\Delta\kappa$ is the drift and the δ_t are i.i.d. homoskedastic random variables with $\mathbb{E}[\delta_t] = 0$. The extrapolation simply predicts the expectation value as $\widehat{\kappa}_{t+1} = \widehat{\kappa}_t + \widehat{\Delta\kappa}$. As the error terms are assumed to be normally distributed, one can easily give confidence intervals for these predictors. However, as the extrapolation is only one of many sources of uncertainty in the tables, these intervals are not very helpful per se.

Variants and improvements of the Lee–Carter method found in the literature include the linear and bilinear models as well as the LC2 model [14] (which includes also the second term of the singular value decomposition), different ARIMA(p,d,q) time series techniques for the extrapolation of the trend and a Poisson fitting [4]. The latter method does not attempt to directly fit the death probabilities, which are not homoskedastic random variables, but rather fit the actual death numbers. Using a Poisson assumption [3] on the numbers of deaths, an approximation of a similar form to the Lee–Carter method can be obtained by a maximum-likelihood fit to the observed deaths.

Using the yearly tabulated death probabilities, the Lee–Carter method (with a normalization of Z using the empirical variance of a previous SVD) was applied to Austrian population mortality. The resulting trends are shown in Figure 7, the corresponding estimated parameters $\widehat{\alpha}_x$, $\widehat{\beta}_x$ and $\widehat{\kappa}_t$ are shown in Figure 8. The range of years in the underlying data was chosen from 1972 to 2002 as discussed in Section 4.5.3.

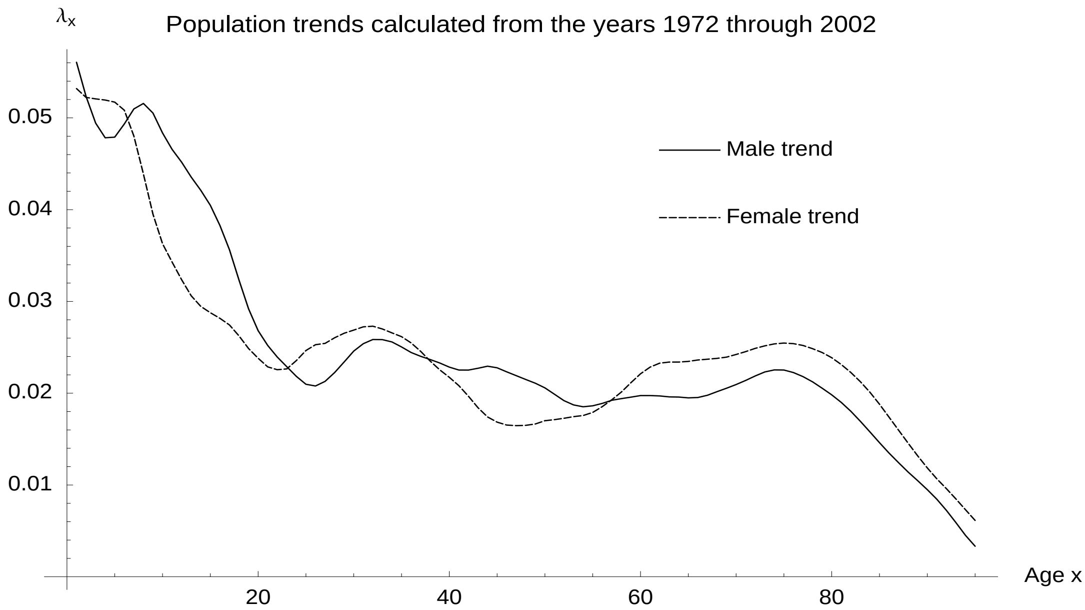


Figure 7: *Austrian population mortality trends, obtained by the Lee–Carter method from the data of the years from 1972 to 2002. The corresponding parameters are given in Figure 8.*

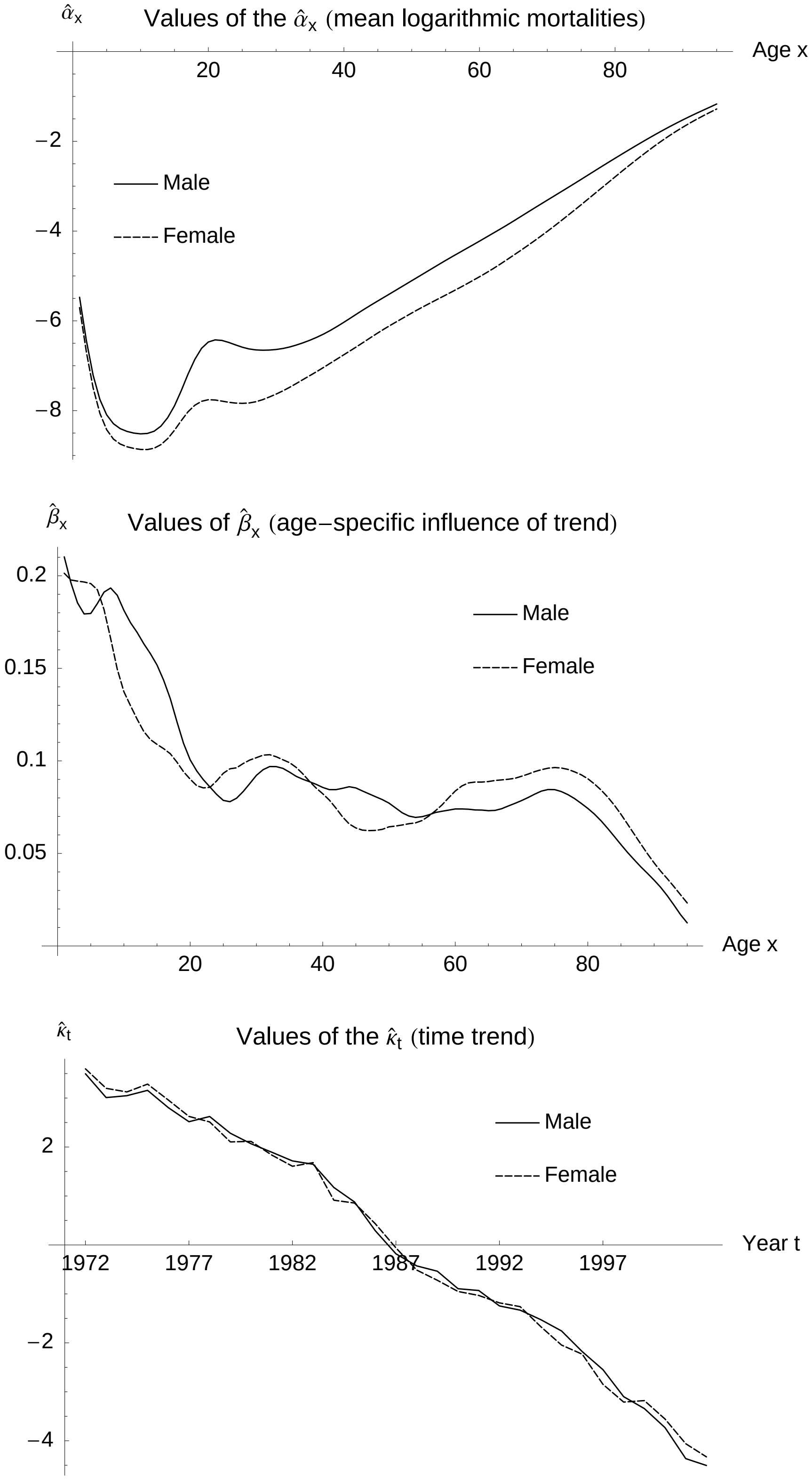


Figure 8: *Estimated parameters of the Lee–Carter decomposition applied to Austrian population mortality data from 1972 to 2002. The variance of old ages is normalized using the empiric variance obtained by a previous non-normalized decomposition.*

Rewriting the Lee–Carter decomposition and using the ARIMA$(0, 1, 0)$ time series extrapolation for κ_t reveals after taking expectations that the estimator for the yearly trend λ_x of a person aged x is $\lambda_x = \widehat{\beta}_x \widehat{\Delta\kappa}$. In the further derivation of the AVÖ 2005R, we will use this trend and—after some adaptions to annuitants—apply it to the base table derived from the census data. The estimated mean log-mortality $\widehat{\alpha}_x$ of the Lee–Carter decomposition will not be used further.

4.5.2 Theoretical Background of the Lee–Carter Decomposition

For convenience, we will now give a proof that the first term of the singular value decomposition of a matrix $\mathbf{A}$ is the bi-linear estimator $\mathbf{x}\,\mathbf{y}^{\mathrm{t}}$ in the sense of a least-squares fit to the matrix.

Lemma 1. *Let $\mathbf{A} \in \mathbb{R}^{m\times n}$ with $m \geq n$ w.l.o.g. and rank $r \leq n$. Its singular value decomposition is*

$$\mathbf{A} = \mathbf{U}\mathbf{D}\mathbf{V}^{\mathrm{t}} \qquad \text{with } \mathbf{U} \in \mathbb{R}^{m\times m},\ \mathbf{V} \in \mathbb{R}^{n\times n} \text{ and } \mathbf{D} = \begin{pmatrix} \widehat{\mathbf{D}} \\ \mathbf{0} \end{pmatrix},\ \mathbf{D} \in \mathbb{R}^{m\times n},\ \widehat{\mathbf{D}} \in \mathbb{R}^{n\times n}$$

where $\mathbf{U}$ and $\mathbf{V}$ are orthogonal matrices and $\widehat{\mathbf{D}}$ is a diagonal matrix with non-negative elements $s_1 \geq s_2 \geq \cdots \geq s_r > s_{r+1} = \cdots = s_n = 0$ (the singular values of $\mathbf{A}$).

Then the best bi-linear approximation (in the least-squares sense) of the form $\mathbf{x}\,\mathbf{y}^{\mathrm{t}}$ with $\mathbf{x} \in \mathbb{R}^m$ and $\mathbf{y} \in \mathbb{R}^n$ is the first term of the singular value decomposition

$$\mathbf{x}\,\mathbf{y}^{\mathrm{t}} = s_1 \mathbf{u}\,\mathbf{v}^{\mathrm{t}}$$

where s_1 is the largest singular value of $\mathbf{A}$ with corresponding (not necessarily unique) left- and right-singular vectors $\mathbf{u} = \mathbf{U}_{\cdot,1}$ and $\mathbf{v} = \mathbf{V}_{\cdot,1}$.

Proof. The existence of the singular value decomposition is guaranteed e.g. by [17, Theorem 7.3]. The singular values are the square roots of the eigenvalues of $\mathbf{A}^{\mathrm{t}}\mathbf{A}$. The columns of $\mathbf{U}$ are corresponding eigenvectors of $\mathbf{A}\mathbf{A}^{\mathrm{t}}$ and the columns of $\mathbf{V}$ are corresponding eigenvectors of $\mathbf{A}^{\mathrm{t}}\mathbf{A}$.

We want to minimize the Euclidean (Frobenius) matrix norm of the deviations

$$f(\mathbf{x}, \mathbf{y}) = \left\|\mathbf{A} - \mathbf{x}\,\mathbf{y}^{\mathrm{t}}\right\|_E^2 = \sum_{i=1}^{m}\sum_{j=1}^{n} \left(A_{i,j} - x_i y_j\right)^2$$

over all $\mathbf{x} = (x_1, \ldots, x_m)^{\mathrm{t}} \in \mathbb{R}^m$ and $\mathbf{y} = (y_1, \ldots, y_n)^{\mathrm{t}} \in \mathbb{R}^n$. Since for all $\mathbf{x} \in \mathbb{R}^m$

$$f(\mathbf{x}, \mathbf{y}) = \begin{cases} f\left(\mathbf{0}, (1, 0, \ldots, 0)^{\mathrm{t}}\right) & \text{if } \mathbf{y} = \mathbf{0}, \\ f\left(\mathbf{x}\left\|\mathbf{y}\right\|, \mathbf{y}/\left\|\mathbf{y}\right\|\right) & \text{if } \mathbf{y} \neq \mathbf{0}, \end{cases}$$

it suffices to minimize under the side condition $\left\|\mathbf{y}\right\|^2 = y_1^2 + \cdots + y_n^2 = 1$.

For a matrix $\mathbf{C} \in \mathbb{R}^{m\times n}$ with columns $\mathbf{c}_1, \ldots, \mathbf{c}_n$, orthogonality of $\mathbf{U}$ implies

$$\left\|\mathbf{U}\mathbf{C}\right\|_E^2 = \sum_{j=1}^{n} \left\|\mathbf{U}\mathbf{c}_j\right\|^2 = \sum_{j=1}^{n} \left\|\mathbf{c}_j\right\|^2 = \left\|\mathbf{C}\right\|_E^2$$

and with a similar argument for the orthogonal matrix $\mathbf{V}$

$$\left\|\mathbf{C}\mathbf{V}^{\mathrm{t}}\right\|_E^2 = \left\|\mathbf{V}\mathbf{C}^{\mathrm{t}}\right\|_E^2 = \left\|\mathbf{C}^{\mathrm{t}}\right\|_E^2 = \|\mathbf{C}\|_E^2 \,,$$

hence $\|\mathbf{U}\mathbf{C}\mathbf{V}^{\mathrm{t}}\|_E^2 = \|\mathbf{C}\|_E^2$.

Define $\tilde{\mathbf{x}} = (\tilde{x}_1, \ldots, \tilde{x}_m)^{\mathrm{t}} = \mathbf{U}^{\mathrm{t}}\mathbf{x}$ and $\tilde{\mathbf{y}} = (\tilde{y}_1, \ldots, \tilde{y}_n)^{\mathrm{t}} = \mathbf{V}^{\mathrm{t}}\mathbf{y}$. Then

$$\begin{aligned} f(\mathbf{x}, \mathbf{y}) &= \left\|\mathbf{A} - \mathbf{x}\,\mathbf{y}^{\mathrm{t}}\right\|_E^2 = \left\|\mathbf{U}\mathbf{D}\mathbf{V}^{\mathrm{t}} - \mathbf{U}\tilde{\mathbf{x}}\tilde{\mathbf{y}}^{\mathrm{t}}\mathbf{V}^{\mathrm{t}}\right\|_E^2 = \left\|\mathbf{D} - \tilde{\mathbf{x}}\tilde{\mathbf{y}}^{\mathrm{t}}\right\|_E^2 \\ &= \sum_{i=1}^{n}\Bigg((s_i - \tilde{x}_i\tilde{y}_i)^2 + \sum_{\substack{j=1\\ j\neq i}}^{n} \tilde{x}_i^2\tilde{y}_j^2\Bigg) + \sum_{i=n+1}^{m}\sum_{j=1}^{n} \underbrace{\tilde{x}_i^2\tilde{y}_j^2}_{\geq 0} \,. \end{aligned}$$

Using $\|\tilde{\mathbf{y}}\|^2 = 1$, we get

$$\begin{aligned} f(\mathbf{x}, \mathbf{y}) &\geq \sum_{i=1}^{n}\left(s_i^2 - 2s_i\tilde{x}_i\tilde{y}_i + \tilde{x}_i^2\right) \\ &= \sum_{i=1}^{n}\underbrace{(\tilde{x}_i - s_i\tilde{y}_i)^2}_{\geq 0} + \sum_{i=1}^{n} s_i^2 - \sum_{i=1}^{n}\underbrace{s_i^2}_{\leq s_1^2}\tilde{y}_i^2 \\ &\geq \sum_{i=2}^{n} s_i^2 \,. \end{aligned}$$

This lower bound is attained for $\tilde{\mathbf{x}} = (s_1, 0, \ldots, 0)^{\mathrm{t}} \in \mathbb{R}^m$ and $\tilde{\mathbf{y}} = (1, 0, \ldots, 0)^{\mathrm{t}} \in \mathbb{R}^n$, hence $\mathbf{x} = \mathbf{U}\tilde{\mathbf{x}} = s_1\mathbf{u}$ and $\mathbf{y} = \mathbf{V}\tilde{\mathbf{y}} = \mathbf{v}$ minimize $\|\mathbf{A} - \mathbf{x}\,\mathbf{y}^{\mathrm{t}}\|_E^2$. □

4.5.3 Period for the Trend

The trend for the AVÖ 2005R is obtained as the mid-term trend since 1972 from the yearly adjusted life tables of Statistics Austria. The old Austrian annuity valuation table AVÖ 1996R in contrast used the short-term trend since 1980 for the years until 2000, after which the trend was assumed to decline to the long-term trend since 1870.

The choice of employing the mid-term trend since 1972 can be justified by several reasons:

- Around the year 1970, a significant change in the trend behavior happened as can already be expected from Figure 9.
- Carter and Prskawetz [5] investigate the Austrian mortality development using the Lee–Carter method and show that the observed change is statistically significant. Therefore, including data from before might turn out problematic as this includes effects that no longer apply to the Austrian population.
- During the last one or two decades, an even increasing trend could be observed to values well above the long- and even the mid-term trend since 1972. This effect can be seen in the time series for κ_t in Figure 8 as well as in the yearly trends shown in Figure 9.

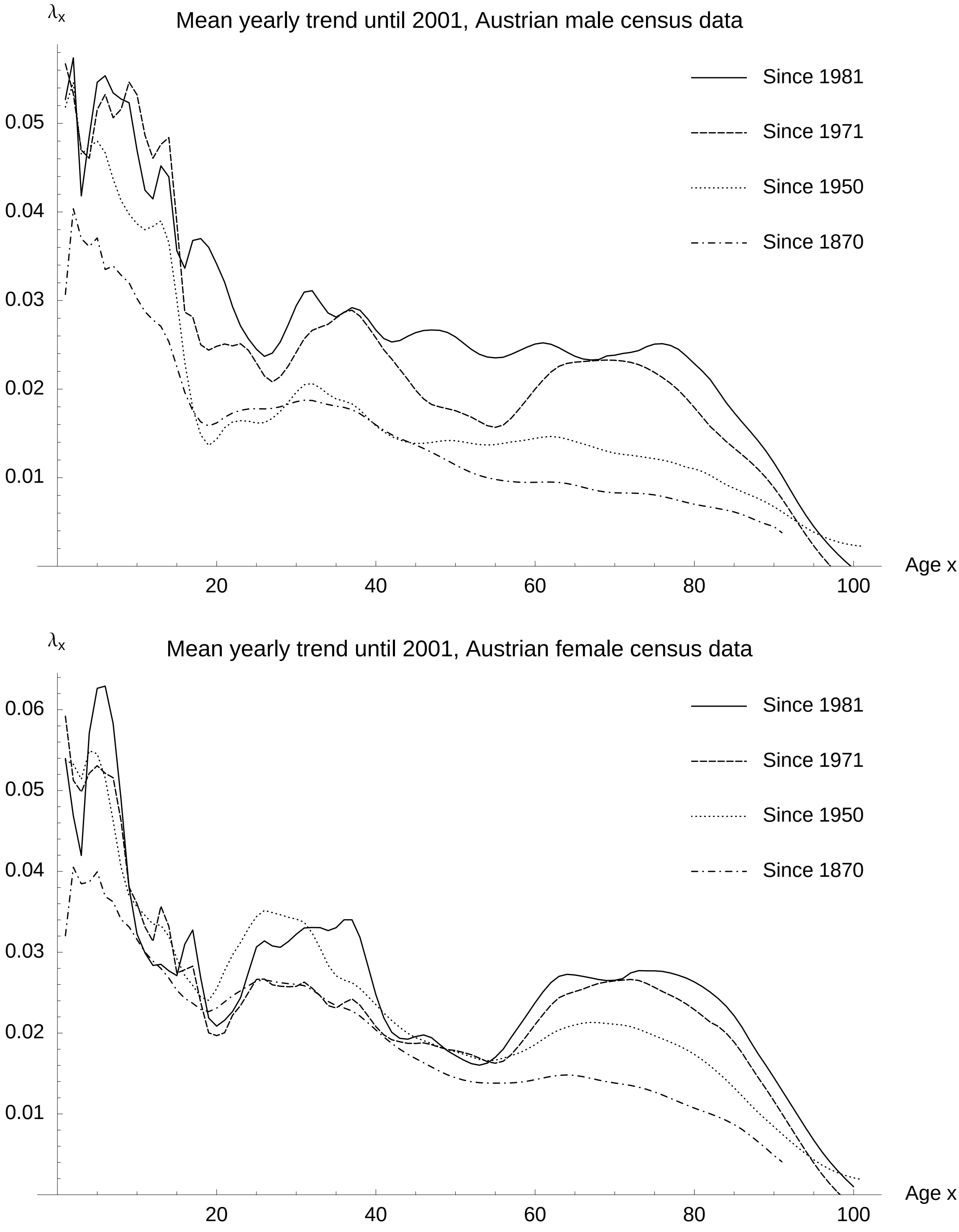


Figure 9: *The short- and mid-term trends of the Austrian population are significantly higher than the long-term trend since 1870.*

- The year 1972 is chosen such that the period ranges over 30 years, but does not include the year 1970 of the last flu epidemic (which might skew the trend considerably). In addition to the requirement of the Lee–Carter method to operate on data from sufficiently many points in time, the latter reason was also one of the arguments to use the yearly raw mortality of Statistics Austria instead of the census life tables.

The Swiss tables ERM/F 1999 use the annuitant data of the periods 1961/65 until 1991/95 to determine the trend by fitting their λ_x.

The German DAV 2004-R also uses a similar time period for their trends, where the trend was obtained from the yearly life tables since 1971/73 to avoid the skewed data of the flu epidemic 1969/70. Moreover, Statistics Austria employs trends from this 30+ years period in several of their population models[11].

4.6 Modifications of the Second-Order Trend

4.6.1 Selection Effects

The Swiss annuitant data indicate selection effects not only on the base mortality, but also on the yearly mortality reduction. In particular, for the ERM/F 1999 the trend parameter λ_x for annuitants (individual contracts) is almost twice as large as the population trend for males and above the population trend for females by a factor of 1.1 to 1.2 [11]. Such a discrepancy in the mortality trend between different socio-economic groups is also asserted by international studies (e.g. in [19] with data from England, Wales, the four northern European countries, Spain and the USA).

In Germany, a comparison of the trends with the compulsory social security also indicates a different trend between blue- and white-collar workers. While this effect is inherent in the Swiss table, the DAV 2004-R accounts for this by an additive term of 0.002 (0.2% additive) in the trends for both males and females.

Due to the lack of Austrian data, the AVÖ 2005R also uses a selection effect of similar magnitude, which roughly corresponds to the factor 1.1 observed in the Swiss data for females. For males, however, this factor seems small compared to the Swiss data[12]. This will be partially compensated by an additional security margin to the trend (see Section 4.7).

In contrast, the previous Austrian table AVÖ 1996R uses the raw population trend for the extrapolation of annuitant mortality without any selection effects applied.

4.6.2 Long-Term Trend

Contrary to the previous Austrian annuity valuation table AVÖ 1996R and the second-order table of the new German table DAV 2004-R, the AVÖ 2005R does not include a decline of the high mid-term trend to the lower long-term trend over the last century.

[11] Private communication with Alexander Hanika of Statistics Austria.

[12] However, when comparing Austrian data with Swiss data, one always has to keep in mind that the Swiss annuity structure is different to Austria, as private annuity contracts have a far longer and more important tradition in Switzerland than in Austria.

This is motivated by multiple reasons:

- The Austrian population data shows a fundamental change in the trend around the year 1970 (as discussed in [5]), which can also be seen in the 20-year mean trends shown in Figure 10. It seems unreasonable to include previous data in the forecast, an approach which is also chosen by Statistics Austria in several of their population forecasts.
- The census data show that the trend has not slowed down in the past few years, but has rather increased (Figure 10).
- While the German DAV 2004-R table includes a trend decline in the second-order table (also in reference to the AVÖ 1996R), in the first-order table this decline is left out as a security margin.

However, a perpetual extrapolation with a high constant trend leads to vanishing death probabilities and thus unreasonably high life expectancies in the far future. While this does not pose a problem for the daily use of the annuity valuation table, it is an inherent weakness of a model with a constant trend. To avoid this issue, we thus propose a long-term trend reduction, which leads to a reasonable limiting life expectancy and non-vanishing limiting death probabilities in the limit $t \to \infty$, while on the other hand it has little influence on the net single premiums and the yearly premiums of annuities calculated from the table AVÖ 2005R.

Instead of using a constant age-dependent mortality reduction factor λ_x per year, we introduce a trend reduction that non-linearly reduces the λ_x to zero as time tends to infinity. The reduction of $\log q_x(t)$ at time t is modelled as

$$\lambda_x(t) = \frac{\lambda_x}{1 + \left(\frac{t-2001}{t_{1/2}}\right)^2} =: R(t)\lambda_x$$

with a half-time of $t_{1/2} = 100$ years. This parameter $t_{1/2}$ determines the speed of the long-term reduction and is defined as the time when the yearly trend has slowed down to half its initial value. Figure 11 shows this time-dependent reduction factor $R(t)$ for the period from 2000 to 2150.

The cumulated trend for the period from the base table 2001 to the year t is obtained by simple integration as

$$G(t)\lambda_x = \int_{2001}^{t} \lambda_x(t)\,dt = \lambda_x t_{1/2} \arctan\left(\frac{t-2001}{t_{1/2}}\right) \quad \text{with } t_{1/2} = 100.$$

Without a long-term trend reduction, the cumulated trend would be $\lambda_x \cdot (t - 2001)$, so the trend reduction can also be seen as a time scaling

$$(t - 2001) \mapsto t_{1/2} \arctan((t - 2001)/t_{1/2})\,.$$

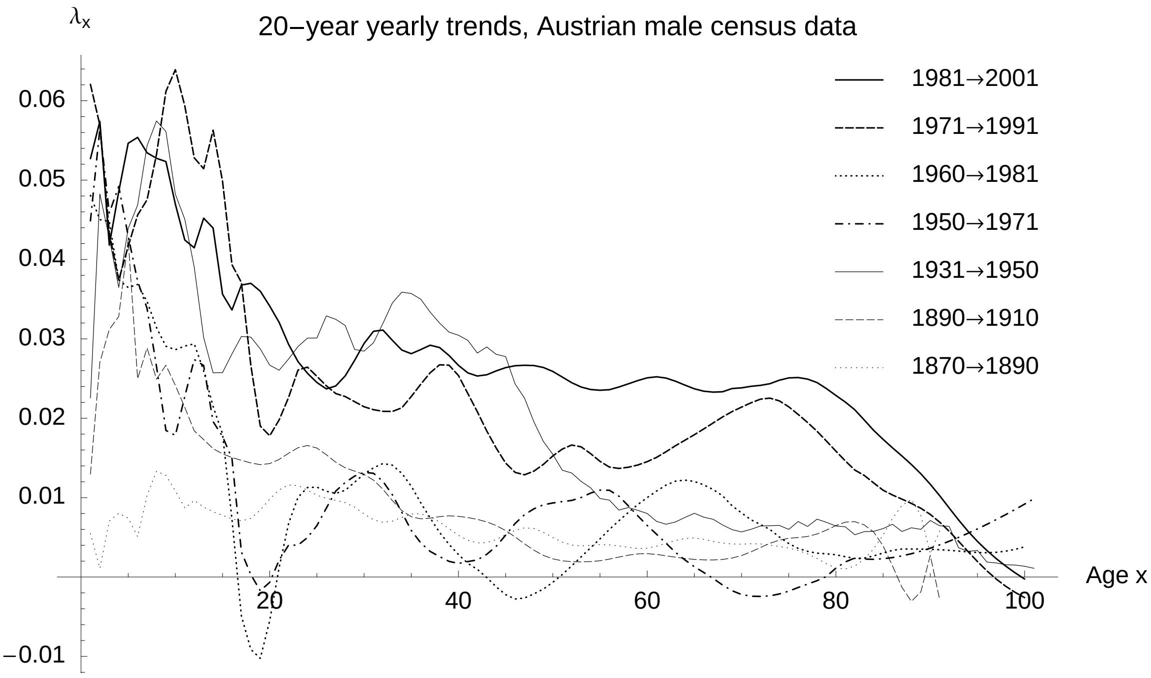
The infinite-time limit of the cumulated trend is

$$\lim_{t\to\infty} G(t) = t_{1/2}\frac{\pi}{2} = 50\pi \approx 157.08,$$

so that the small, yet finite, limiting death probabilities are calculated as

$$q_x^{\text{lim}} = q_x^{\text{base}}(2001) \exp\left(-50\pi\lambda_x\right)\,.$$

As one can see, these are the same probabilities as forecasted for the year 2158 without a trend decline.



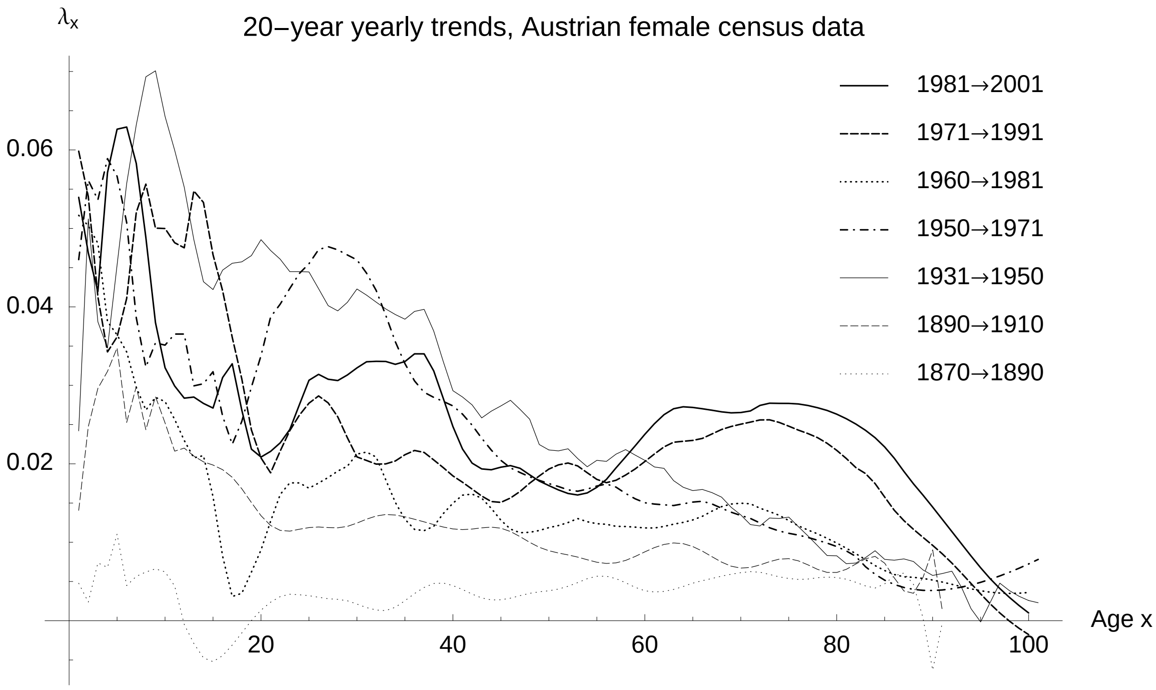


Figure 10: *The 20-year mean trends also show that around 1970 the trend increased dramatically for old ages and continues to increase.*

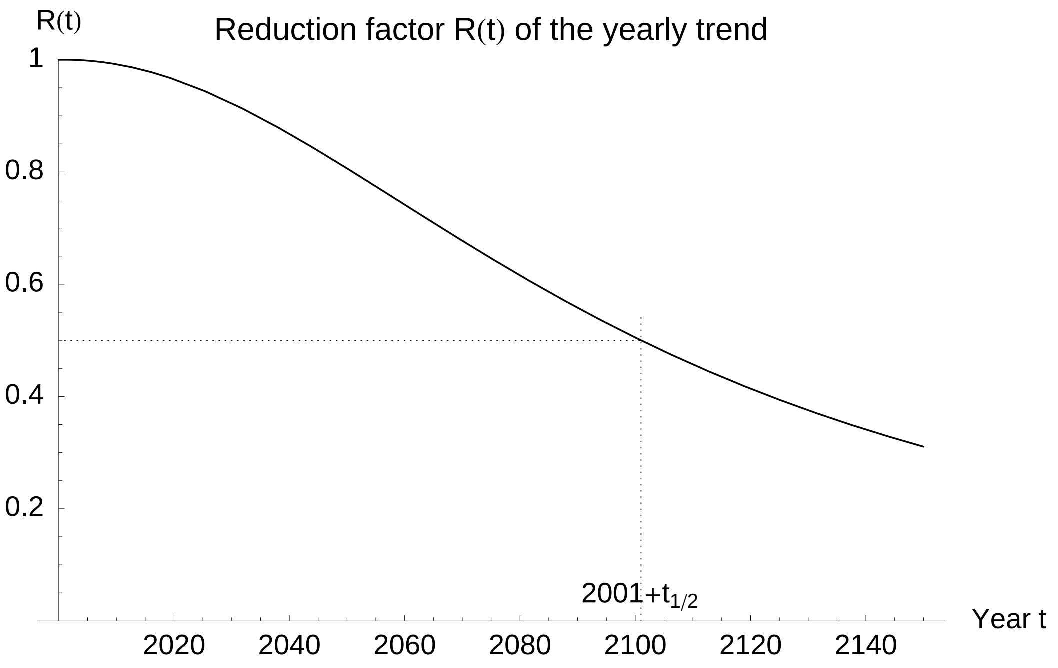


Figure 11: *The trend in calendar year t is reduced by a factor $R(t)$, in $t_{1/2} = 100$ years it is reduced to half its initial value.*

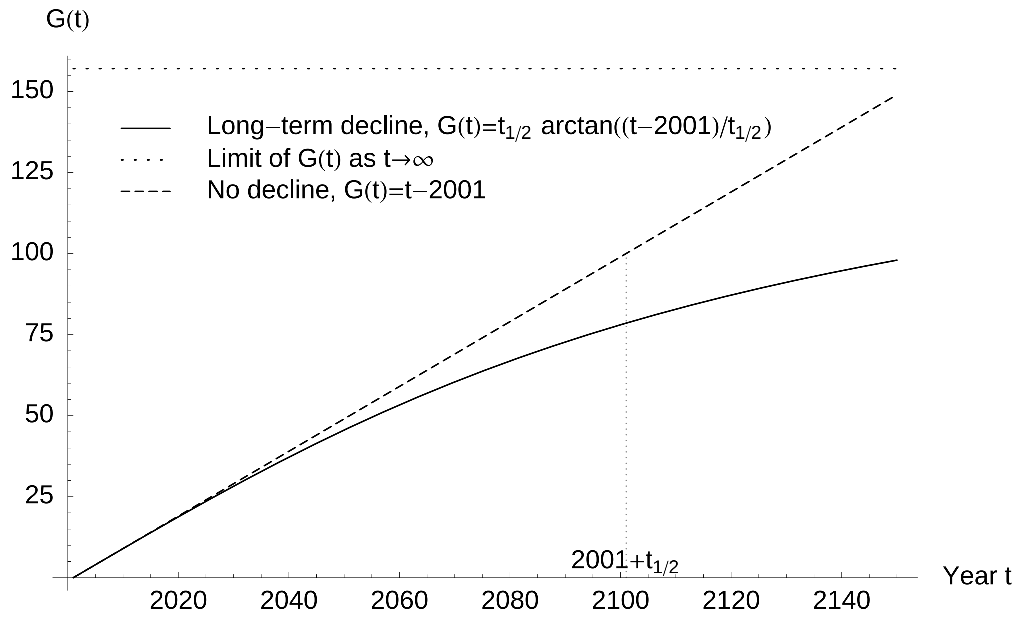


Figure 12: *Cumulated trend $G(t)$ compared to linear extrapolation (i.e. no decline).*

4.7 Security Margins and Adjustments for the First-Order Trend

The adjustments to the population trend described above generate the actual trend for Austrian annuitants in the future (2nd-order trend). However, their derivation is only an actuarial best estimate under several assumptions, based on the available past data.

4.7.1 Security Margins

In particular, there are several forms of risk in the tables:

- Model risk: The annuitant mortality might not have the structure and development as described in the model above. This also includes the possibility that the mortality development might change at some future point of time to a non-log-linear development, or that the mortality development displays heavy cohort effects.
- Parameter risk: Even if the model is chosen correctly, the parameters are only fitted from the available data and might thus be inaccurate. Moreover, because some aspects like the selection factors are partly derived from German values, they might be inappropriate for the Austrian annuitants.
- Risk of random fluctuations: The tables only predict the mean value over a large population of annuitants. As the typical annuity portfolio of an Austrian insurance company is quite small, the random deviations from the mean values might be considerable. There will be no security margins included in the tables for this type of risk. A detailed discussion of how to deal with it will be presented in a subsequent article.

In the tables developed in the previous sections, there are several concrete factors that introduce adverse model and parameter risk. The most important are:

- The base life table and the applied selection factors might be inaccurate. As the selection factors are adjusted to the German values, they might not be entirely appropriate for Austrian annuitants and underestimate the selection effects. (parameter risk)
- The population group used to generate the table might not coincide with the future group of annuitants. For example, in the future different selection effects might prevail, or the "average" group of annuitants (for which this table was determined) is not representative for the target group of a particular insurance company. (model risk)
- The trend includes a selection effect which only takes social selection into account, so this selection factor on the trend probably underestimates the actual selection. (parameter risk)
- The data used to obtain the values in the table are affected by statistical fluctuations, so the determined empirical parameters might not be accurate.
- The trend was obtained by a least-squares fit of the years 1972 to 2002, where the later years display a higher trend than the years at the beginning of that period (see

Figures 8 and 9). Thus, the current and future trends might be underestimated. (model risk)

- The trend, albeit already high, might even increase further in the near future, due to advances in medicine, in particular for old ages. (model risk)

For all these reasons, a certain security margin needs to be added to account for these risk factors in the determined tables. Consequently, the AVÖ 2005R increases the trend for both males and females by a constant additive term of 0.003, while the base table does not include any security margin. This margin on the trend has a similar effect on annuities starting in 2005 as a 10% security margin on the base table.

This approach has the advantage that the security is actually increasing with time, rather than diminishing when the margin is added to the base table.

The German table DAV 2004-R in contrast adds a security margin of 10% (factor 0.9 on the death probabilities) on the base table. Additionally, the trend in the DAV 2004-R is increased additively by 0.0025, a value which is obtained by a simulation of one further random trend increase in the next 100 years. While we do not assume a short-term trend decline in our second-order tables, the German second-order tables assume such a decline, which is then left out in the first-order tables as an additional security margin.

4.7.2 Trend for Old Ages

The increase of the trends in the past few decades was particularly dramatic for ages between 60 and 90 years (Figures 9 and 10), which unfortunately coincides with the age range with the greatest influence on annuity premiums. This age range is also most responsible for the changes of the premiums compared to the old table AVÖ 1996R.

For both males and females, the census data indicate that this hump is moving further towards higher ages. This is also in line with the medical advances in geriatrics, which result in a considerable mortality reduction at this age range. There is no indication that these advances will slow down considerably in the near future. Consequently, for the AVÖ 2005R this hump is shifted towards higher ages by 5 years. This is done by inserting 5 years with a constant high trend at the maximum of the hump and moving all higher ages by 5 years.

While for ages above 100 years virtually no mortality reduction has been observed in the past few decades, it cannot be guaranteed that this will also be the case in the next few decades. For this reason a slower, exponential decline towards zero,

$$\lambda_x = c \exp\left(-\frac{x-100}{d}\right) \qquad \text{for } x \geq 100\,, \tag{13}$$

is used as the trend for old ages. The corresponding parameters are listed in Table 3. Figure 13 shows the effect of these two trend adjustments for old ages.

In comparison, the German table DAV 2004-R adds a lower bound of 1% for the trend, which leads to unreasonably low death probabilities for ages between 110 and 120 years. The Swiss table ERM/F 1999, in contrast, also extrapolates the trend as tending towards zero. What is remarkable in the Swiss table is that the male trends are much larger than

	c	d
Males	0.00731235	7.20672
Females	0.01001410	8.90257

Table 3: *Parameters of the trend extrapolation to ages x above 100 years.*

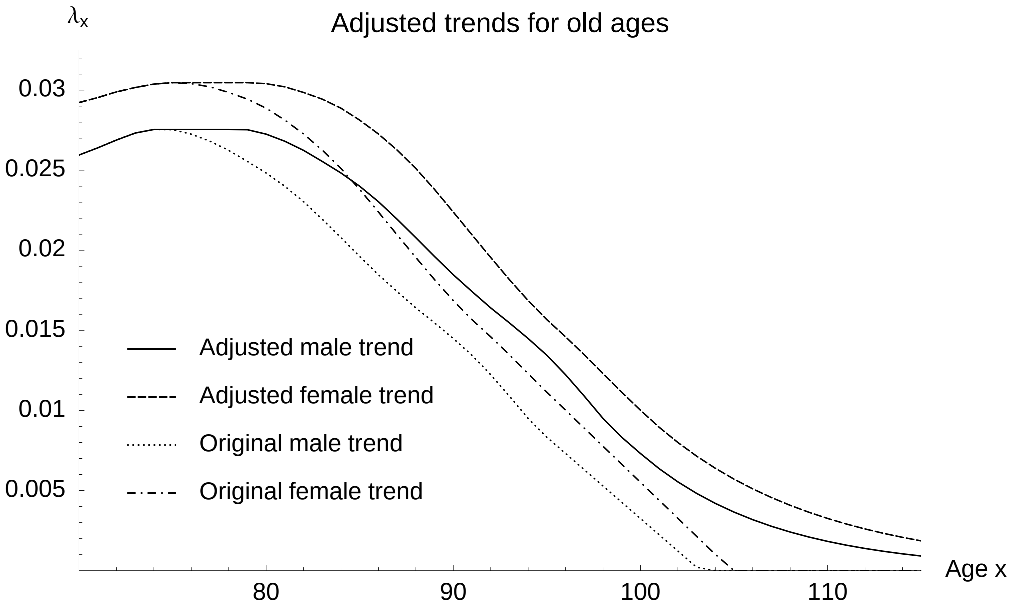


Figure 13: *Trend adjustment for old ages: (1) the maximum of the trend hump is extended by 5 years and all older ages are shifted by 5 years; (2) starting from age 100, the trend decline is slowed down and tends towards zero exponentially instead of linearly.*

the female trends, while the Austrian census data and the German tables exhibit roughly the same magnitude of the trends, with a slightly higher trend for females.

For very young ages below 10 or 15 years, the raw trend obtained from the data is well above 5% mortality reduction per year. These values display large fluctuations (between 4% and 6% per year), and an extrapolation with such a trend quickly leads to non-monotone death probabilities for very young ages. For these reasons, we follow the German approach and introduce an upper cut-off for the trend of 0.05.

4.7.3 Monotonicity

The hump in the trend for old ages also poses another problem: For ages between 40 and 60 years, the mortality reduction is smaller than for ages above 60 years (Figure 7). Using these trends for extrapolation, even for the generation 2005 the death probabilities would no longer be monotone for females. Figure 14 shows the predicted death probabilities if the raw trend with the hump and the resulting pit in the age range from 40 to 60 years is used. The effect for females is far larger, since the pit is more distinct (see Figure 7).

This non-monotonicity could lead to non-monotone premiums for some types of life annuity contracts. For this reason, the trend is interpolated linearly between the ages of 21 and 75 years for males, and between 18 and 63 years for females (see Figure 15). For males, the end of the interpolation coincides with the maximum of the trend, so the whole trend is monotone and non-monotone death probabilities can never occur. For females, the same approach would lead to an unreasonable increase of the trends. Thus, only a large part (but not all) of the non-monotonicity in the trend is cut off. Even though these trends are not monotone, the death probabilities will remain monotone in the final limiting life table.

Austrian annuity contracts typically grant a premium refund on death during the accumulation period, so the death probabilities before payments start have a very limited influence on the net present value and the premiums of such contracts. As the linearization described above mostly affects the accumulation phase of life annuities, it does not have much influence on actual values, although it considerably changes the trend and thus the death probabilities for ages below 60 years. Additionally, the death probabilities in these age ranges are already very low, so even a considerable change of the $q_x(t)$, $x < 60$, cannot significantly change the net single premium of an immediate annuity-due.

4.8 Trend of the AVÖ 2005R After All Modifications

Figure 16 shows the trend employed in the AVÖ 2005R with all modifications applied as discussed in the previous sections[13]. For comparison, the short- and long-term trends of the AVÖ 1996R are also shown in the plot.

One can clearly see the large differences to the AVÖ 1996R for both male and female annuitants. The most important factor is the hump for ages between 60 and 95 years. In this age range, the second-order yearly mortality reduction is well above even the short-term trend assumed in the AVÖ 1996R: For males the largest difference of 0.0053 is at age 76, while for females the largest difference of 0.0077 appears at 77 years. With the additional corrections and security margins, the maximum trend difference of the first-order tables compared to the AVÖ 1996R is 0.0097 at age 80 for males and 0.012 for females aged 81.

Another noteworthy fact is that the long- and short-term trends for females according to the previous table AVÖ 1996R were not so far apart as the trends for males. Since the AVÖ 2005R does not include a trend decline any more, the change affects females less than males.

4.9 Confidence Intervals

In the previous sections, actuarial best estimates were derived from statistical data of the Austrian population. As the original data clearly includes random fluctuations, a proper statistical investigation of the error, in the form of confidence intervals on the parameters and the resulting death probabilities, would be in order. Confidence intervals for the extrapolation of the time series for the κ_t can easily be given due to the well-understood structure of the time series, but confidence intervals for the other parameters can only be obtained by a bootstrap simulation, which perturbs the initial data stochastically,

[13] The trend increase for old ages (see Section 4.7.2) is understood in these plots and in the final table as a first-order effect, although we expect it to be a second-order effect to a large extent.

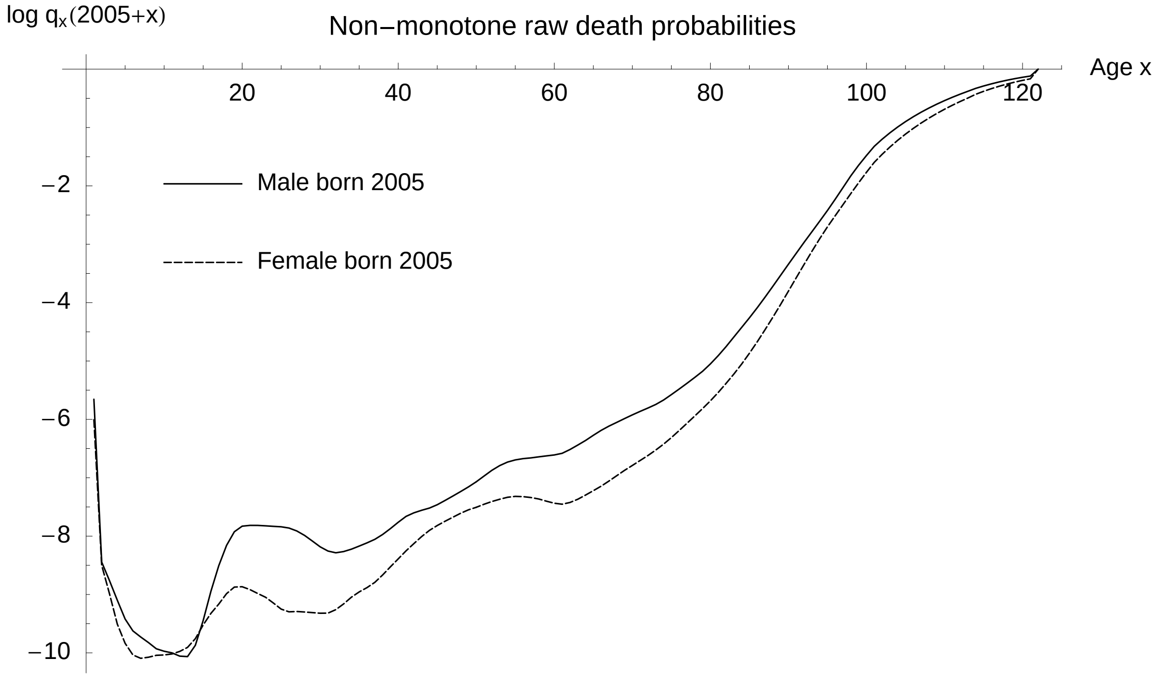


Figure 14: *Extrapolating the death probabilities with the raw trends leads to non-monotone female death probabilities above age 40 already for the generation 2005.*

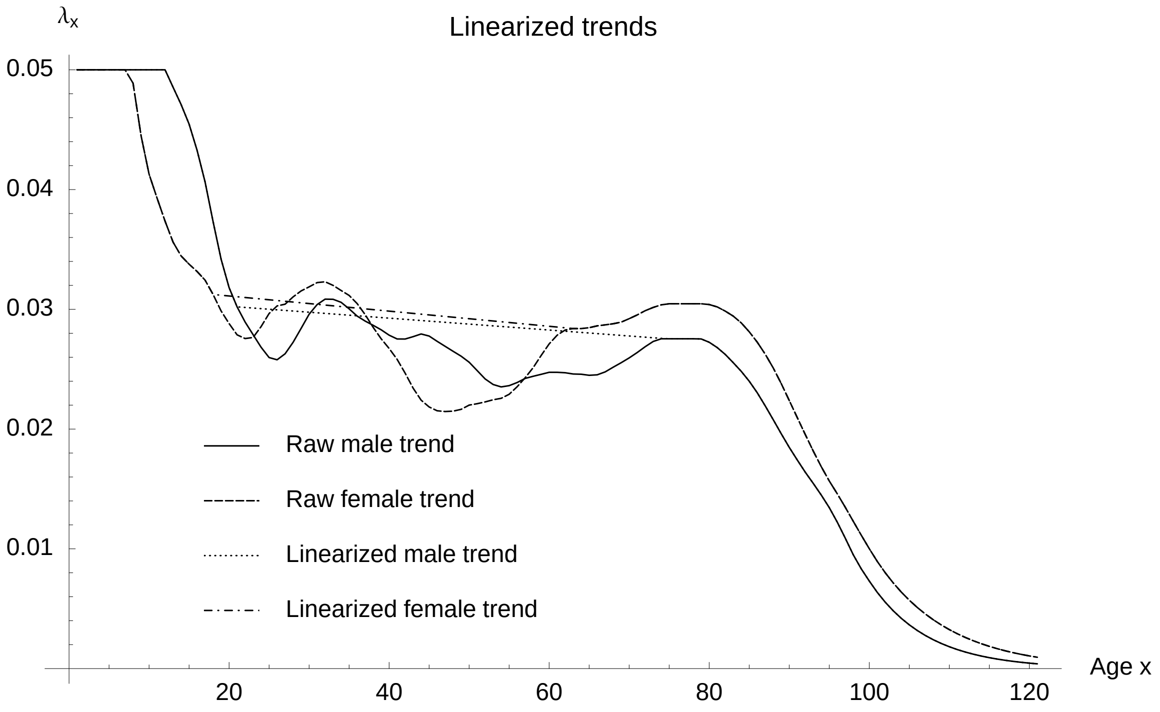


Figure 15: *A linear interpolation of the trend is applied to ensure monotone projected death probabilities in the next decades.*

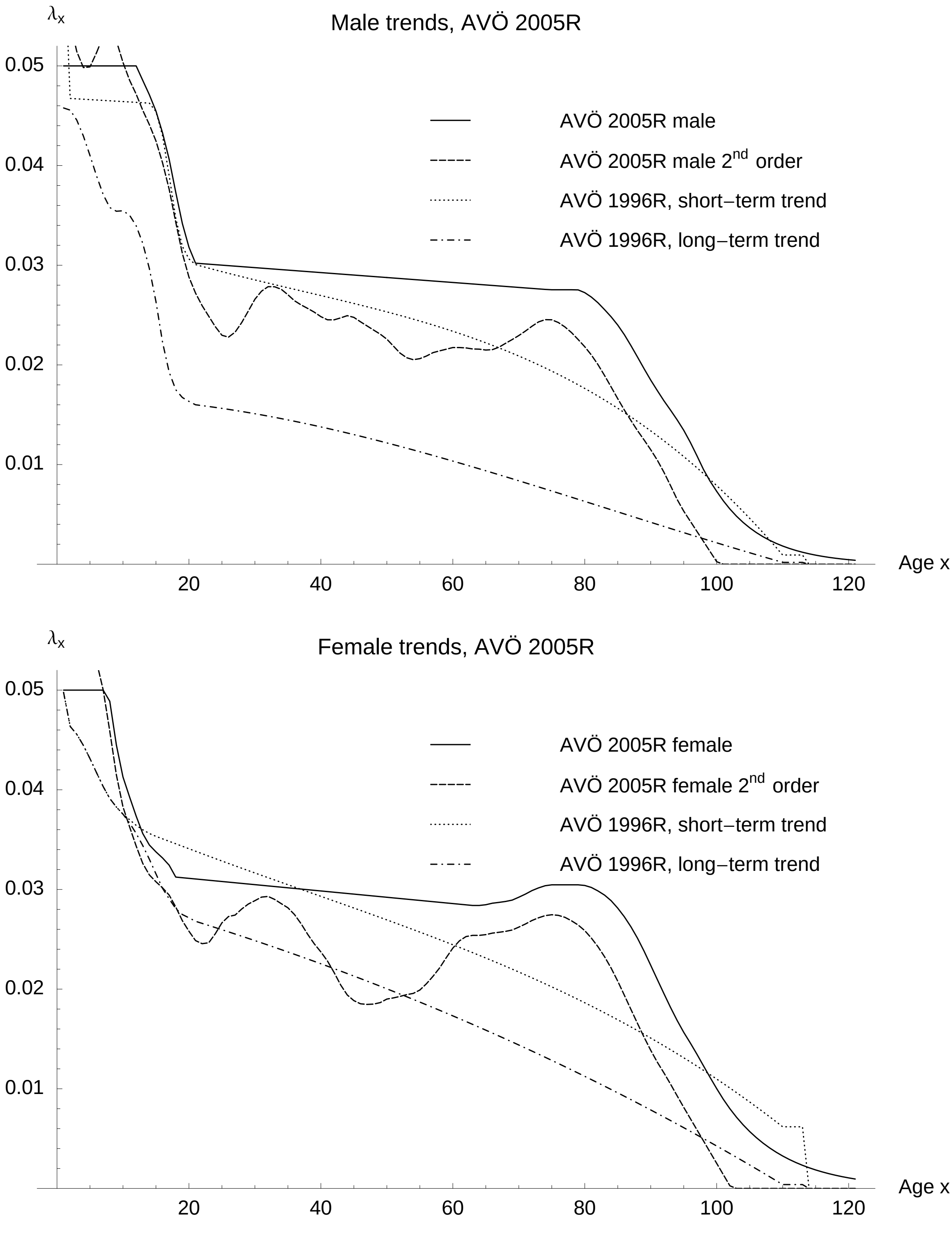


Figure 16: *Trends of the AVÖ 2005R after all modifications, compared to the short- and long-term trends of the AVÖ 1996R.*

derives new parameters with these perturbed initial values and then derives confidence intervals from these parameters. Furthermore, in lack of Austrian annuitants data, several assumptions for the selection were made, so it might be hard to give concrete confidence intervals for the resulting death probabilities.

A proper investigation of confidence intervals for the various pieces of the derivation might be the topic of a future diploma thesis at the department.

4.10 Age Shifting for the AVÖ 2005R

The AVÖ 2005R table presented in the previous sections is a two-dimensional life table $q_x(t)$, where the death probabilities depend on the age x as well as on the calendar year t. Although we strongly encourage every company to use the exact table presented above and discourage anyone from using such an approximation, we will still propose an approximated one-dimensional table for convenience, which reproduces the most common net single premiums of the AVÖ 2005R to a certain (but limited) degree.

To generate this one-dimensional approximation to the exact table, we employ the method of an age shift (as presented in Section 2.4), where the death probabilities are described by one reference table $q_x^{\mathrm{AS,base}}(\tau_0)$ for the reference birth year τ_0. The mortality, or rather the actuarial values, for other generations with birth years $\tau = t - x$ are approximated by using the values from the reference table

$$\ddot{a}_x(t) \approx \ddot{a}_{x+\Delta(\tau)}^{\mathrm{AS,base}}(\tau_0)$$

with an adjusted age $x + \Delta(\tau)$ with $\Delta(\tau_0) = 0$. In the sequel, we will simply write $\ddot{a}_x^{\mathrm{AS,base}}$ instead of $\ddot{a}_x^{\mathrm{AS,base}}(\tau_0)$.

The time period in which the AVÖ 2005R will be applied to new annuity contracts is intended to be roughly from 2006 to 2015 and the age for new contracts is typically between 30 and 60 years (or 25 to 65 years). If we take the mean of these two ranges, the death probabilities of the generation $\tau_0 = 1965$ form a suitable base table for the age shift, as determined by the two-dimensional exact annuity valuation table. For ages below 35 years, the death probabilities are taken directly from the observations of Statistics Austria, with the selection factors of Section 4.3 applied. As the death probabilities for young ages are increasing as well as decreasing (accident hump), the base table needs to be monotonized to ensure a monotonic age shift. This is done according to the backward recursion

$$q_x^{\mathrm{AS,base}}(1965) = \begin{cases} 1 & \text{for } x = 121, \\ \min\Big(q_x(1965 + x), q_{x+1}^{\mathrm{AS,base}}(1965)\Big) & \text{for } x = 120, 119, \ldots, 0. \end{cases}$$

The values $q_x(1965+x)$ are the death probabilities of the generation 1965 according to the exact table for $x \geq 35$ and the observed values for $x < 35$ with selection factors applied. The maximum age was chosen at $\omega = 121$ years. The resulting base table is shown in Figure 17 and tabulated in Appendix A.6.

As a one-dimensional mortality curve can certainly not reproduce all results calculated with the exact two-dimensional table, the shift $\Delta(\tau)$ is obtained as a weighted mean over

Figure 17: *Base table of the age shift for the generation 1965.*

the shifts required to reproduce immediate life annuity-dues as good as possible. Most other actuarial values can be derived from them.

For a given birth year τ, define for each age x the age shift $\Delta_x(\tau)$ for the net single premium of an immediate annuity-due relative to the base table as

$$\Delta_x(\tau) = \widehat{x} - x + \frac{\ddot{a}_{\widehat{x}}^{\text{AS,base}} - \ddot{a}_x(\tau + x)}{\ddot{a}_{\widehat{x}}^{\text{AS,base}} - \ddot{a}_{\widehat{x}+1}^{\text{AS,base}}}\,.$$

The integer part $\widehat{x}$ of the adjusted age is obtained as the (unique) index fulfilling

$$\ddot{a}_x(\tau + x) \in \left(\ddot{a}_{\widehat{x}+1}^{\text{AS,base}}, \ddot{a}_{\widehat{x}}^{\text{AS,base}}\right]\,.$$

In both equations, $\ddot{a}_x(t)$ is the net single premium of an immediate annuity-due according to the exact table. The over-all age shift $\Delta(\tau)$ for the generation born in year τ is then calculated as the weighted mean[14] of the $\Delta_x(\tau)$, i.e.

$$\Delta(\tau) = \sum_{x=x_{\min}}^{x_{\max}} \frac{w_x}{\sum_{\widetilde{x}=x_{\min}}^{x_{\max}} w_{\widetilde{x}}} \Delta_x(\tau)\,, \tag{14}$$

[14]The age shift of the AVÖ 1996R was similarly obtained as the average of the ages $x = 55, 60, 65$ and 70 years. The DAV 2004-R in contrast calculates the age shift as the uniform mean over annuities starting in the years 2005 until 2020. For annuities with a long deferral time (i.e. for young ages x), such an approach can lead to significant errors, as the relevant quantity to determine the premiums in this case is not $\ddot{a}_x(2005)$, but rather the net single premiums at the time when annuity payments start.

with

$$x_{\min} = \max\{50, 2005 - \tau\}$$
$$x_{\max} = \max\{90, 2010 - \tau\}$$

and with weights w_x as given in Table 4. The general age range considered for the approximation is 50–90 years. However, the lower summation index for birth years τ between 1920 and 1955 is adjusted to $2005 - \tau$ so that only present and future annuity values are included in the average. For birth years prior to 1920, the upper and lower bounds of the summation are adjusted so that only the next five years are used to fit the age shift for very old ages.

Age x	50–59	60–70	71–90	> 90 years[15]
Weight w_x	1	5	3	1

Table 4: *Weights for the mean of the age shifts.*

The choice of weights puts most emphasis on the age range 60 to 70 years, which is when annuity payments typically start (since most contracts offer a refund of premium if the insured dies before the payments start, the earlier ages are almost irrelevant). Ages from 71 to 90 years are weighted only a little less, since this is the age range where the insurance needs to keep reserves for the remaining future payments. The age range 50 to 59 was only included with small weight to account for other contracts where payments start earlier. When only these net single premiums are needed, the approximation is best; for all other values derived from the annuity valuation table, the discrepancy to the correct value might be considerable.

Interestingly, for early birth years the age shift decreases again, which would lead to increasing net single premiums and reserves for persons born earlier. To avoid this undesired effect, the final age shift is monotonized again.

Another fact to note is that the age shift is obtained by approximating certain annuity net single premiums, which depend on the interest rate that is used. Thus, also the age shift is interest-dependent in general, so an interest rate of 2.25% was used for the shift of the AVÖ 2005R. However, using an interest rate of 4% does not significantly change the shift, only for very few birth years a different age shift would be obtained.

4.10.1 A Note on the Quality of the Approximation via Age Shifting

Rueff [16, Section III.2] already notices that the quality of an approximation via age shifting is limited by the degree with which the actual mortalities can be transformed into each other. Thus, the approach depends on the slope and curvature of the log-mortality staying roughly constant. The best fit is possible when the log-mortalities $\log q_x$ are linear in x. However, due to the high trend of the age interval 60 to 80 years and the subsequent low trend for higher ages, this is no longer the case to a satisfactory degree in the AVÖ 2005R.

Looking at the actual development of the net single premium of an annuity-due for a male aged 60 and 80 (Figure 18) shows the consequence of this non-linearity: While the

[15] The age range $x > 90$ is only included for very old ages, i.e. birth years prior to 1920.

exact table for age 60 increases faster than the approximated table, it increases slower for age 80. Figure 19 shows a quantification of this effect by comparing the approximated values with the values according to the exact table. As the approximated future values are far lower than the exact values, the current values of the age shifted table need to overestimate the current exact values to balance future errors. As a consequence, the age shifted table leads to current reserves that are far larger than actually required.

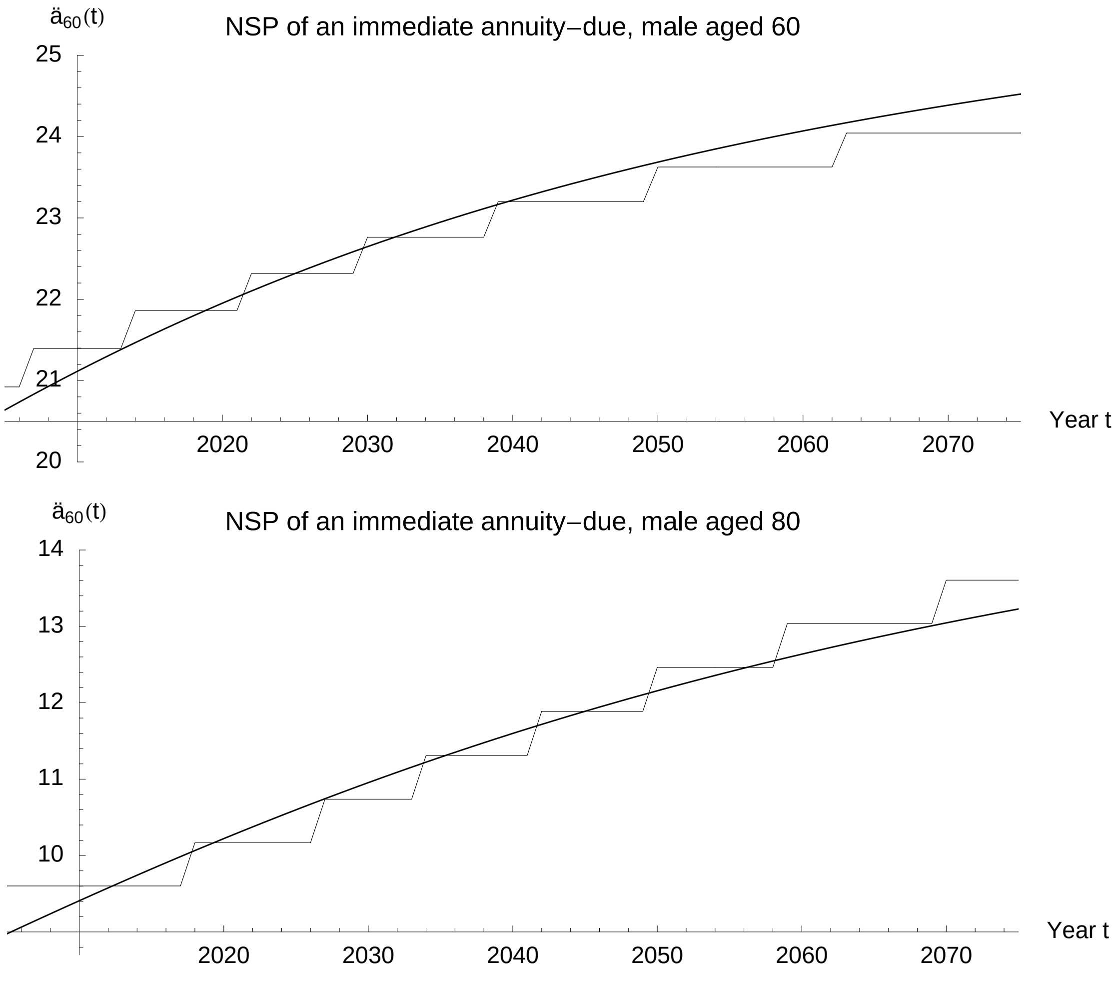


Figure 18: *Comparison of the net single premiums of the exact and the age shifted table. For age 60, the exact table increases more, while for age 80 the age shifted table has a higher increase.*

5 (International) Comparisons

5.1 Base Table with Selection

Figure 20 shows an international comparison of the period life tables for the year 2001, which is the base year for the extrapolation of the AVÖ 2005R. While all three new annuity valuation tables (AVÖ 2005R, DAV 2004-R and ERM/F 1999) lie essentially below the extrapolated annuity valuation table of the AVÖ 1996R for the year 2001, they agree to a large extent in particular for the important age range of 60 to 85 years. One has to notice,

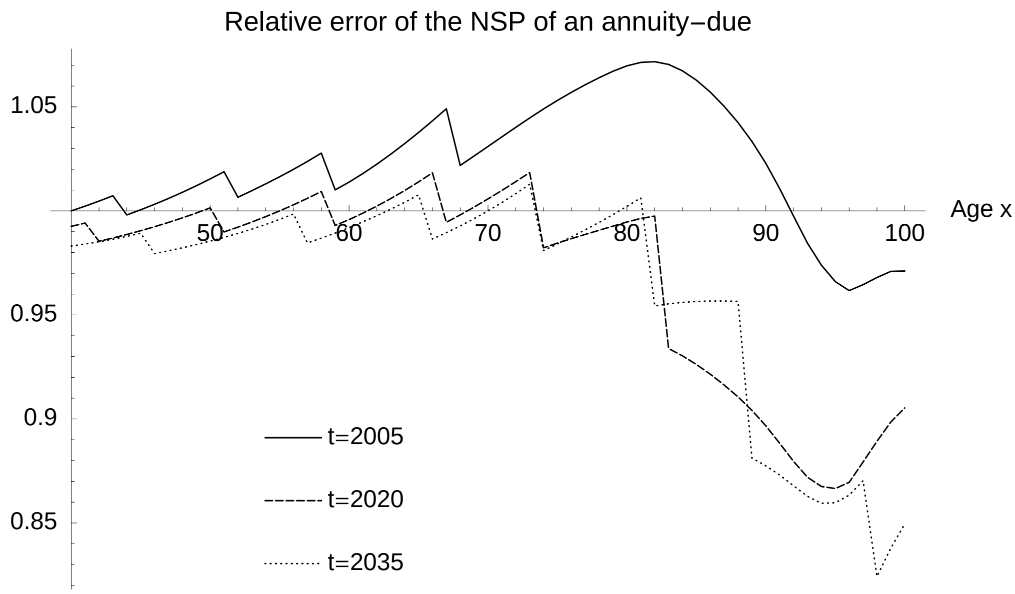


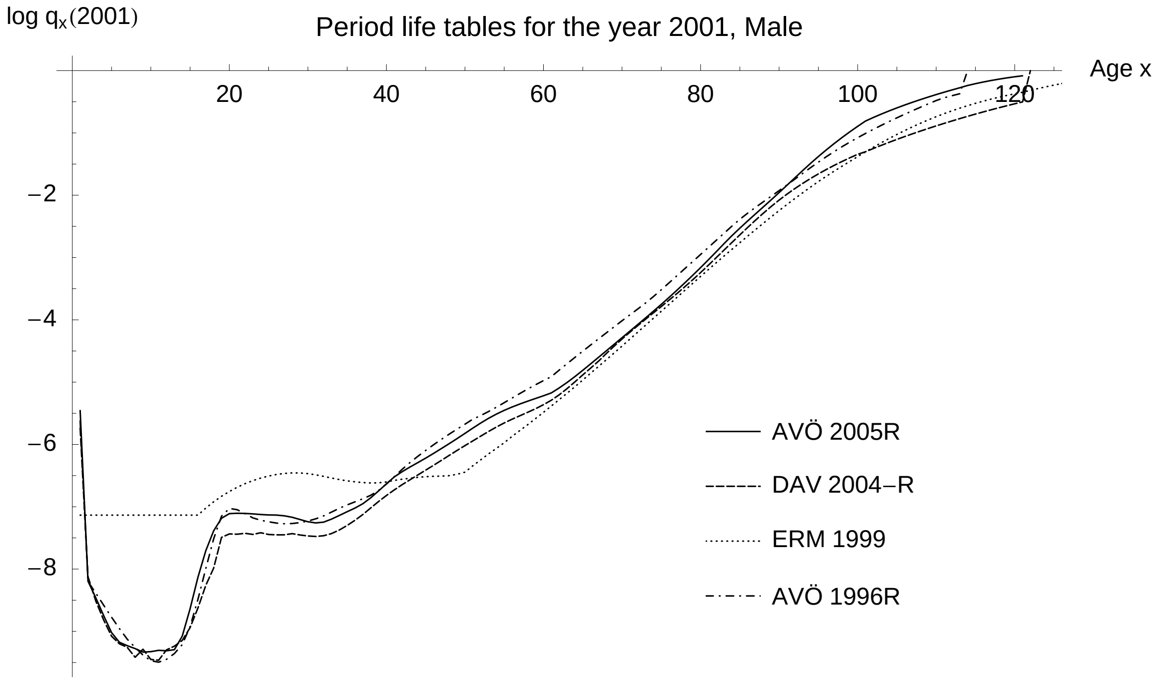
Figure 19: *Relative error* $\ddot{a}^{\mathrm{AS,base}}_{x+\Delta(t-x)}/\ddot{a}_x(t)$ *of the age shifted table, compared to the values of the double-graded exact table. In the future, the approximation leads to values far lower than the exact table, thus current net single premiums and reserves are overestimated.*

however, that for very old ages the new AVÖ 2005R lies even above the old table. This stems from the fact that the extrapolated mortality of the official Austrian population life table has even increased for ages above 95 years compared to the death probabilities used for the AVÖ 1996R. In addition, for old ages almost no mortality reduction has been observed, while the AVÖ 1996R assumed a slight trend for these ages. Thus, the projected mortality lies below the actual current value. Another thing to notice is the low mortality level in the German table for years above 95 years, which is a consequence of the extrapolation method used in the DAV 2004-R. This is one of the reasons (apart from the fact that no security margins for random fluctuations are included) why the Austrian actuarial values calculated from the new table are considerably lower than the German values.

5.2 Trends

Figures 21 and 22 show the first- and second-order trends of the AVÖ 2005R compared to the trends of the DAV 2004-R, the ERM/F 1999 and the AVÖ 1996R. The most noticeable fact is that the trends of all three new tables are far above the long- and even the short-term trend of the AVÖ 1996R.

Although they were obtained from different data and using different fitting methods, the shapes and magnitudes of the Austrian and German trends agree to a high degree. This is one more indication that the population structure in Austria and Germany is very similar and that the methods employed for the DAV 2004-R might also be used for the Austrian table. The main difference is the age range above 100 years, where the German second-order table uses an artificial lower bound of $\lambda_x \geq 0.01$, which is then shifted by



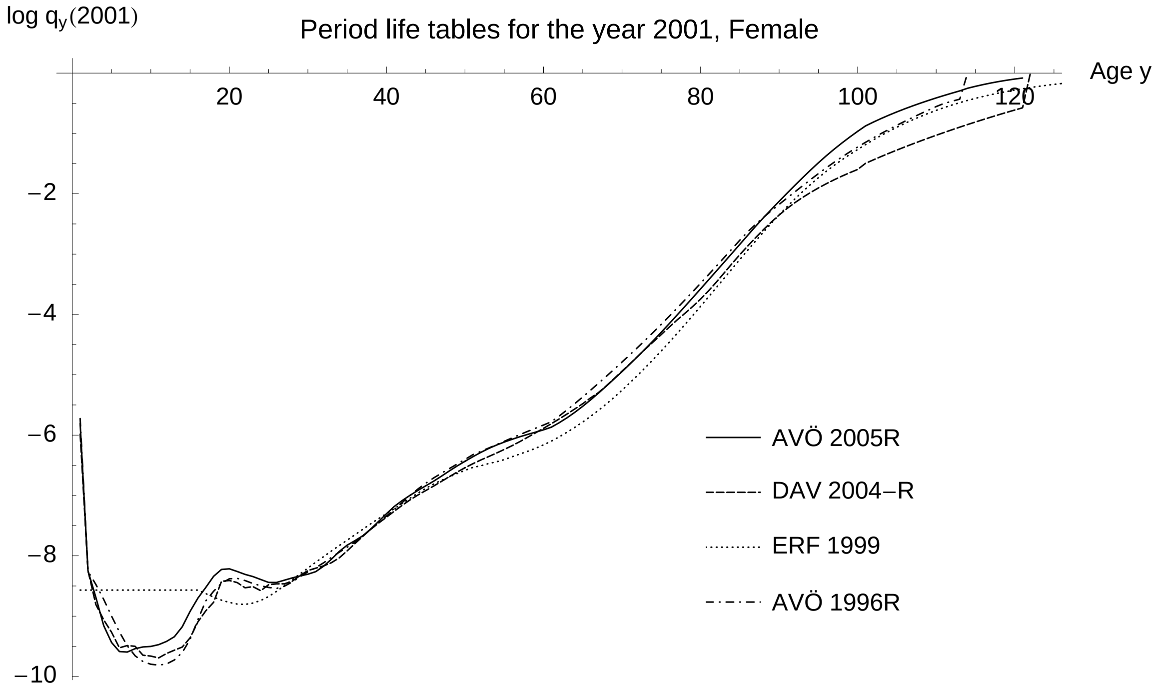


Figure 20: *Comparison of the base table of the AVÖ 2005R (based on the year 2001) with the previous Austrian and the current German and Swiss annuity valuation tables for the year 2001. While the German and Swiss tables include an additional term for the risk of random fluctuations, the Austrian table does not.*

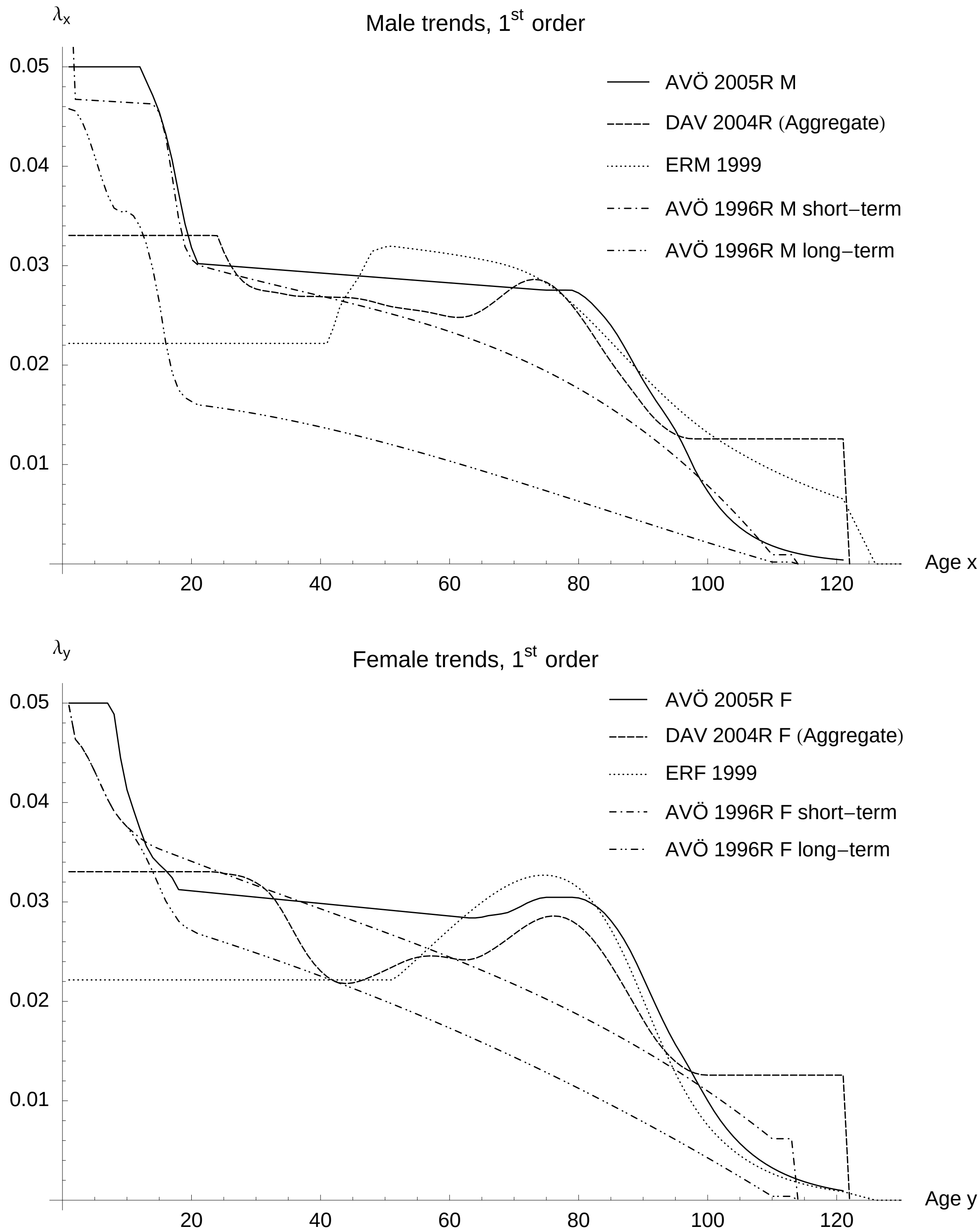


Figure 21: *First-order trends of the AVÖ 2005R compared to the corresponding trends of the DAV 2004-R, ERM/F 1999 and the AVÖ 1996R.*

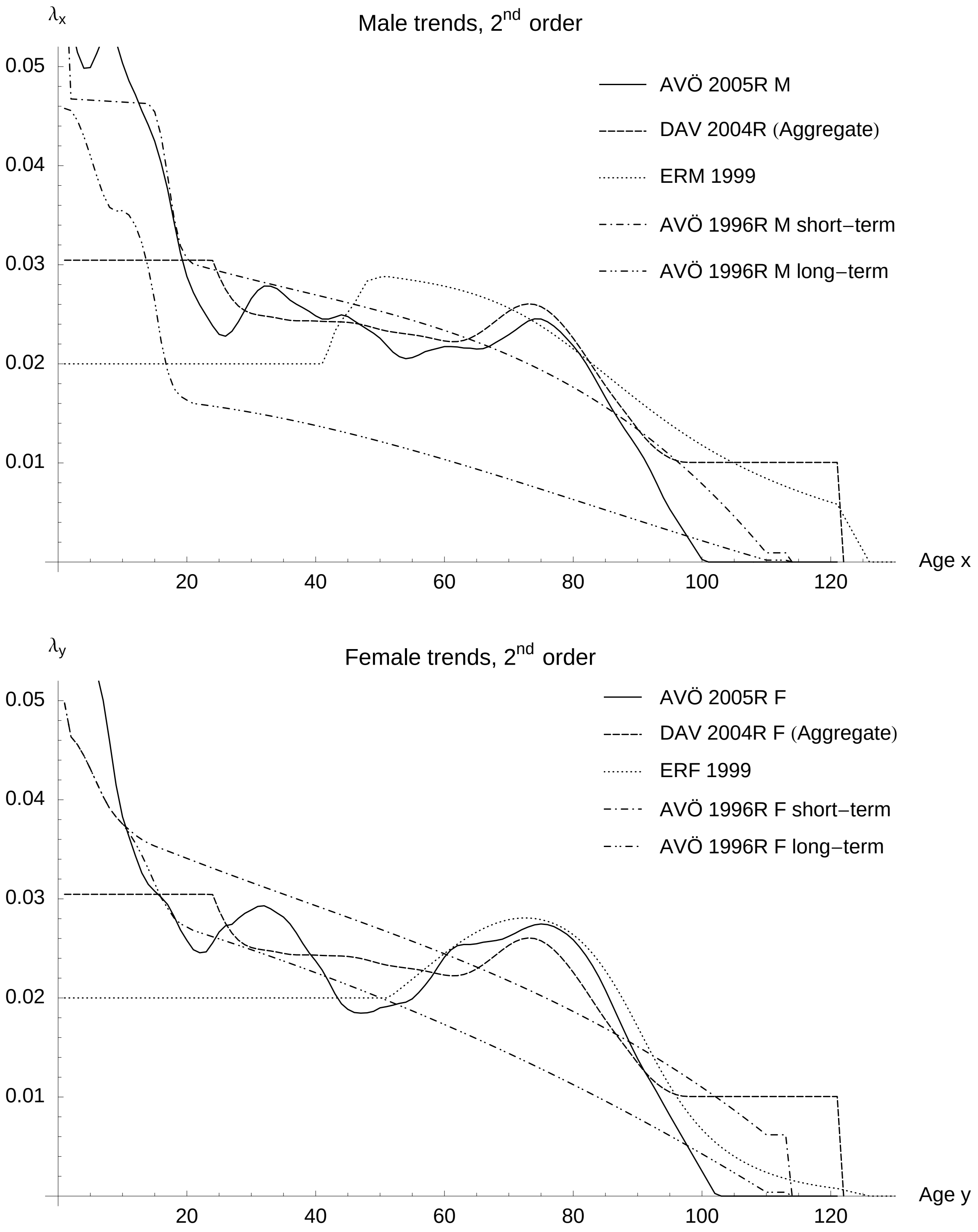


Figure 22: *Second-order trends of the AVÖ 2005R compared to the corresponding trends of the DAV 2004-R, ERM/F 1999 and the AVÖ 1996R.*

another security margin for the first-order trend. In our view, this approach adds an unnecessary large security margin for old ages, in particular as no mortality reduction for this age range could be observed throughout the last 50 years. The new Austrian table, in contrast, lets the trend approach zero, which—at least for females—also agrees with the Swiss trend. The male trend for old ages in the Swiss table is also considerably higher than the Austrian trend. To quantify the effect of the trend of old ages, we construct life tables similar to the AVÖ 2005R, only with the trend for old ages replaced by the German or Swiss trends. These tables use first-order trends and also employ the long-term trend reduction (or equivalently, the time-scaling) discussed in Section 4.6.2. The results for the net single premiums of a life annuity-due starting at age 65 can be seen in Table 5.

		ERM-trend for $x \geq 100$		ERM-trend for $x \geq 90$		DAV-trend for $x \geq 95$	
	AVÖ 2005R	NSP	rel. change	NSP	rel. change	NSP	rel. change
$\ddot{a}_{65}$ M	21.335	21.456	0.6%	21.742	1.9%	21.611	1.3%
$\ddot{a}_{65}$ F	22.703					22.931	1.0%

Table 5: *Influence of the trends of old ages on the net single premiums of a whole life annuity-due of 1 starting at age 65 for the generation 1990.*

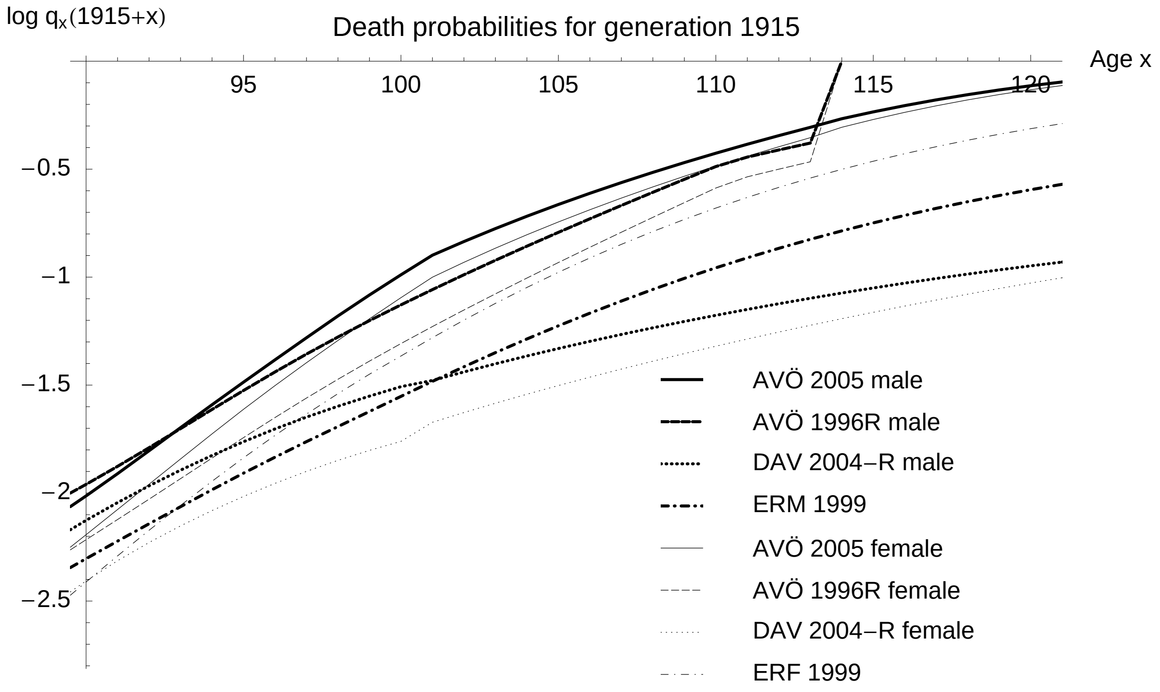
The first column is the NSP of a life annuity-due to a 65-year old person (born 1990, 2.25% interest, NSP at the time when payments start) according to the AVÖ 2005R. Using the higher Swiss trends for males only for ages ≥ 100 years, the relative change is only 0.57%, but using the ERM trends starting already at 90 years, the increase is considerable (1.93% relative change). Moreover, using the German lower bound of 0.01 for the trend plus 0.0025 security margins leads to a substantial increase for males (1.3%). For females, the Swiss trend is slightly lower than the Austrian trend, so we do not compare these. However, the German lower bound on the trend again leads to an increase of 1.00%. While this is less than the males increase, it is still considerable and explains a large part of the difference between the new Austrian and German tables.

To conclude, a large part of the difference in the NSPs calculated from the AVÖ 2005R and the DAV 2004-R can be attributed to the lower bound for the trend in the German table. The Swiss table for males also employs a trend for old ages, which is considerably larger than the Austrian trend obtained by some modifications from the population trend.

The age range above 100 years seems to be of minor importance, but the age range from 90 to 100 years plays an important role. As Figure 16 shows, the first-order trend in the AVÖ 2005R for male annuitants aged 95 is already twice as large as the second-order trend, which in turn is larger than the population trend. Considering this, we conclude that although the male trends of the AVÖ 2005R are well below the Swiss and German trends for old ages, they should be sufficient for Austrian annuitants.

5.3 Mortality

In this section, we compare the resulting death probabilities of the new Austrian table for some selected generations with the corresponding values predicted by the German and Swiss tables. Figures 23 and 24 depict the mortality of the following generations:



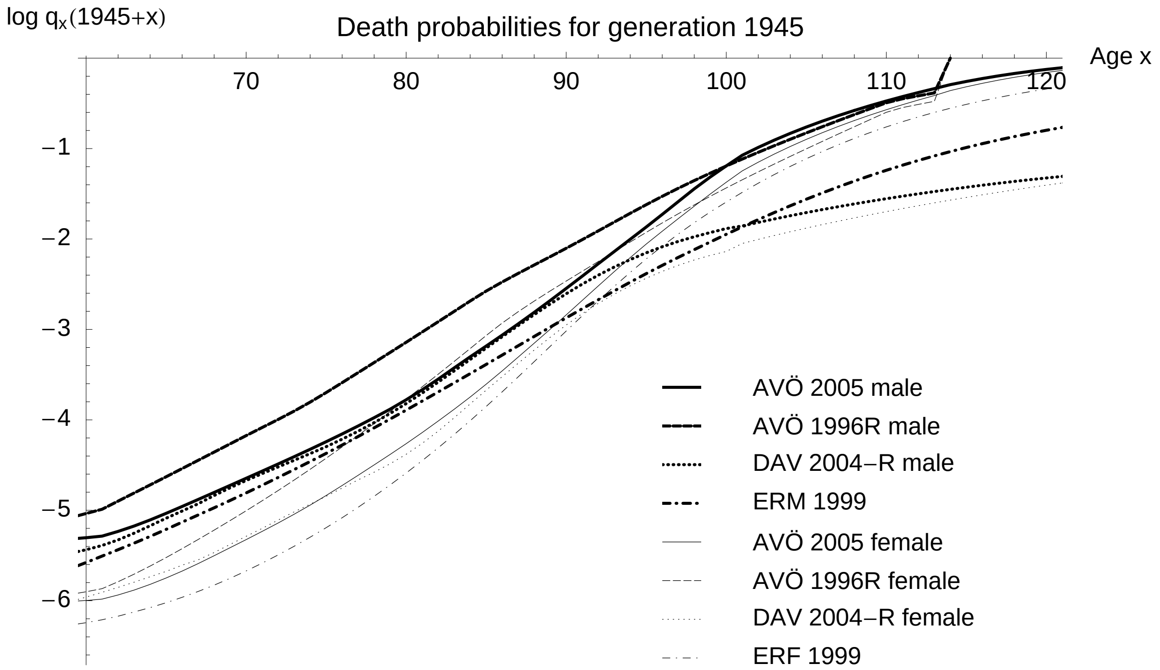
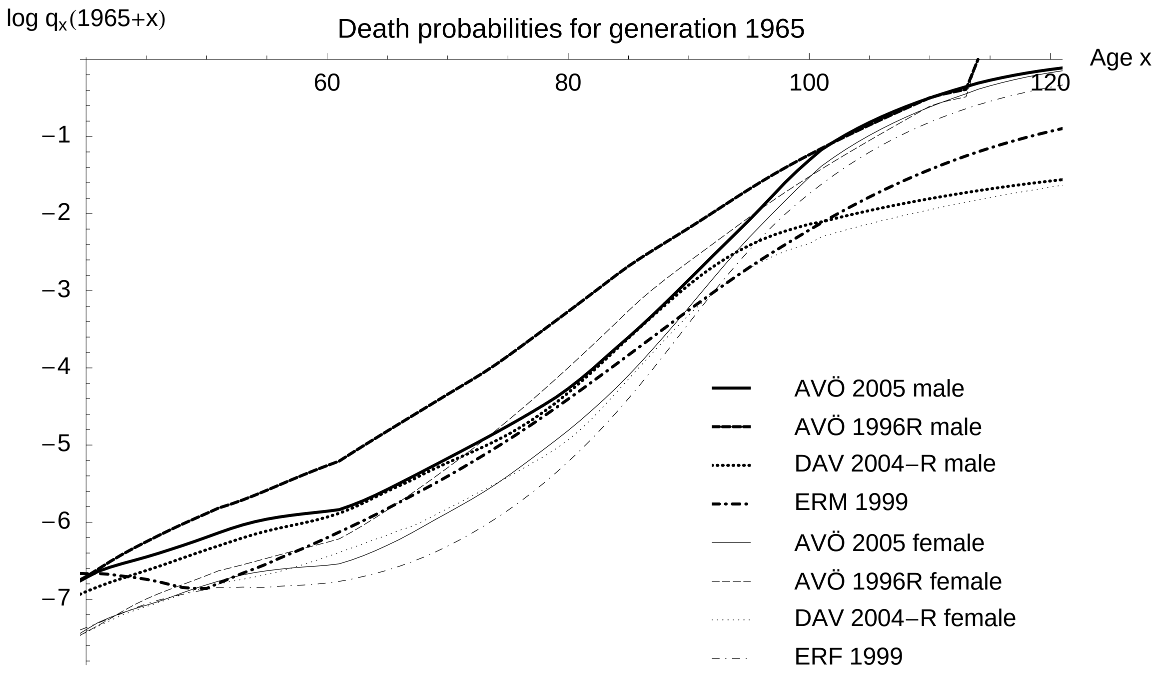


Figure 23: *International comparisons of the logarithms of death probabilities for the generations 1915 and 1945. While the German and Swiss tables include an additional term for the risk of random fluctuations, the Austrian table does not.*



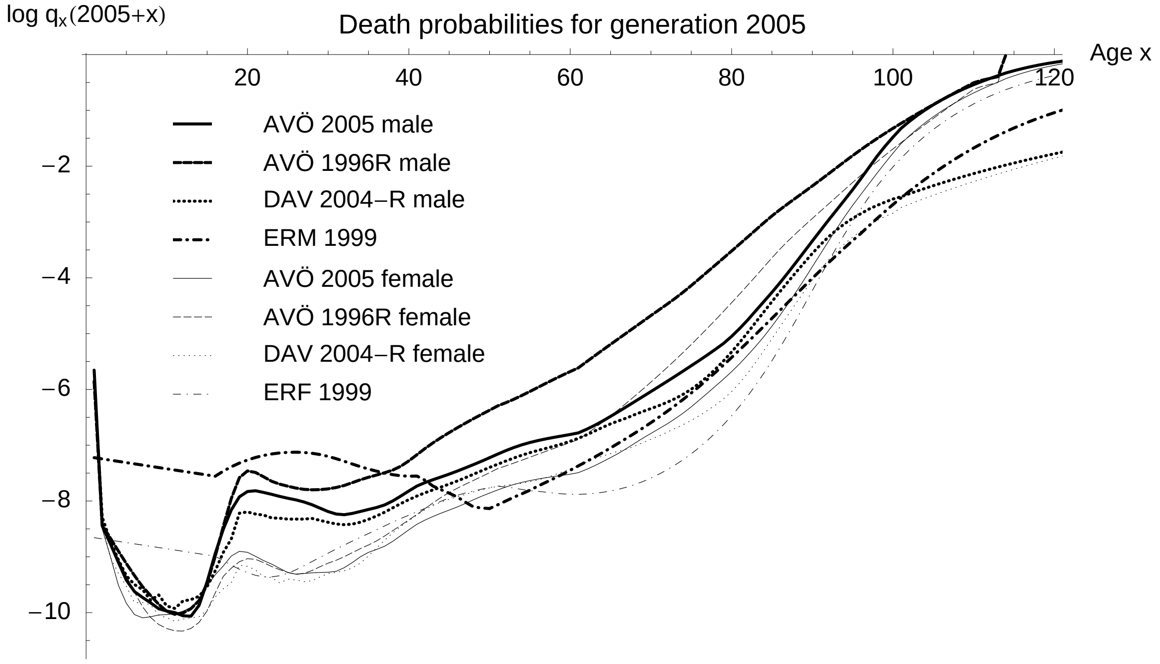


Figure 24: *International comparisons of the logarithms of death probabilities for the generations 1965 and 2005. While the German and Swiss tables include an additional term for the risk of random fluctuations, the Austrian table does not.*

- Generation 1915: Currently aged 90 years, which is at the upper age range relevant for reserves.
- Generation 1945: Currently aged 60 years. This is the typical generation for which annuitization starts during the life span of the AVÖ 2005R.
- Generation 1965: Currently aged 40 years. This is the average generation for new contracts calculated with the AVÖ 2005R.
- Generation 2005: Born now, which is beyond the range of birth years where this table is meant to be applied but it might still be useful to see the future development.

For most age ranges, the AVÖ 2005R predicts slightly higher death probabilities than both the German and the Swiss tables (except for old ages, which is discussed above in detail).

5.4 Life Expectancy for Annuitants

Using the second-order mortality predictions $q_x^{(2)}(t)$ for annuitants, the expected future lifetime $\mathring{e}_x(t)$, the expected curtate future lifetime[16] $e_x(t)$ and the corresponding quantiles for annuitants aged $x \in \mathbb{N}_0$ in the year t can easily be calculated. In particular, we have

$$e_x(t) = \mathbb{E}\big[\lfloor T_x(t) \rfloor\big] = \sum_{k=1}^{\infty} {}_k p_x^{(2)}(t)\,,$$

$$\mathring{e}_x(t) = \mathbb{E}\big[T_x(t)\big] = -\sum_{k=0}^{\infty} \frac{{}_k p_x^{(2)}(t)\, q_{x+k}^{(2)}(t+k)}{\log p_{x+k}^{(2)}(t+k)}\,, \tag{15}$$

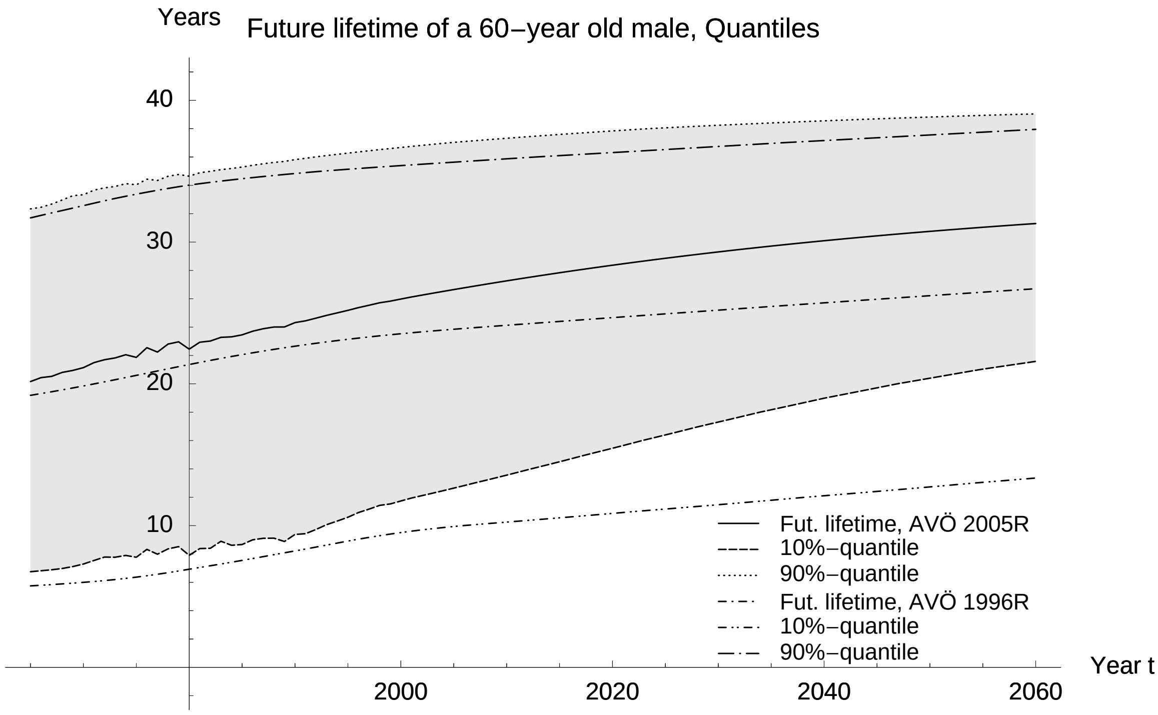
where the random variable $T_x(t)$ denotes the future lifetime of a person aged x in year t and the expression for the expected future life time was derived under the assumption of a constant rate of mortality throughout each year (see Section 5.4.1). Figure 25 shows a comparison with the AVÖ 1996R. While for males the expected future lifetime increases considerably compared to the predictions of the AVÖ 1996R, the increase is much smaller for females. This is again a consequence of the large discrepancy in the trends and the base tables for males between the old and the new table.

Even more interesting are the α-quantiles $u_\alpha(t,x)$ of the expected future lifetime (Figure 25), defined for $\alpha \in (0,1)$ by

$$u_\alpha(t,x) = \min\big\{s \in \mathbb{R} : \mathbb{P}(T_x(t) \le s) = 1 - {}_s p_x^{(2)}(t) \ge \alpha\big\}$$

for a person being alive at age x in year t. We concentrate on $\alpha = 10\%$ and $\alpha = 90\%$.

[16] The random variable $T_x(t)$ of the future lifetime can be split into $T_x(t) = K_x(t) + S_x(t)$, where the curtate future lifetime $K_x(t) = \lfloor T_x(t) \rfloor$ is the number of completed future years lived by a person with age x in year t, while $S_x(t) = T_x(t) - K_x(t)$ is the fraction of the death year before the person dies. The expected curtate future lifetime $e_x(t) = \mathbb{E}[\lfloor T_x(t) \rfloor] = \mathbb{E}[K_x(t)]$ can always be calculated using only yearly death probabilities. The expected future lifetime $\mathring{e}_x(t) = \mathbb{E}[T_x(t)]$, however, can only be calculated using assumptions for the probabilities of death for fractions of a year. One popular assumption is $\mathbb{E}[S_x(t)] = \frac{1}{2}$ and thus $\mathring{e}_x(t) = e_x(t) + \frac{1}{2}$, which is in particular the case when one assumes linearity of $[0,1) \ni u \mapsto {}_u q_{x+k}(t+k)$ for all $k \in \mathbb{N}_0$ with $\mathbb{P}(K_x(t) = k) > 0$ (see Footnote 7 on page 72). In that case, $S_x(t)$ is uniformly distributed in $[0,1)$ and even independent from $K_x(t)$. Under the assumption of a yearly constant force of mortality, however, $\mathbb{E}[S_x(t)] \neq \frac{1}{2}$ and $K_x(t)$ and $S_x(t)$ are in general dependent, even for $x \in \mathbb{N}$ and $t \in \mathbb{N}$. The expression (15) for the future lifetime then becomes less intuitive.



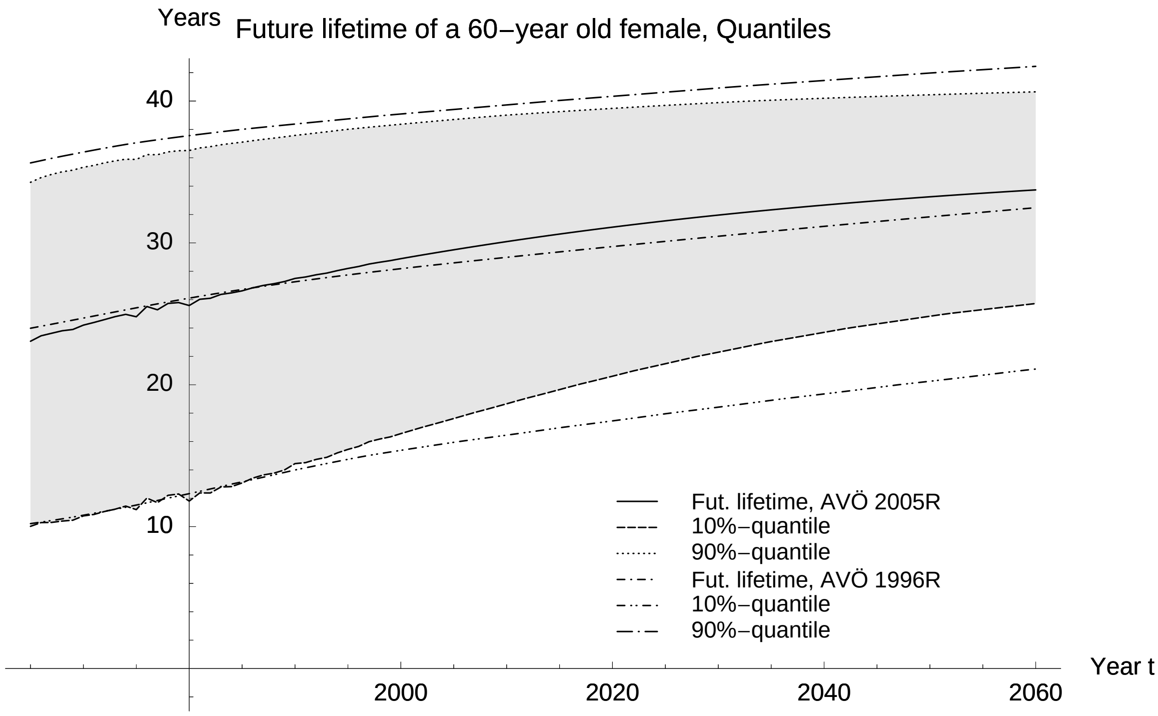


Figure 25: *Expected future lifetime and its 10%- and 90%-quantiles, calculated from the second-order mortality of the AVÖ 2005R and the AVÖ 1996R. The gray area represents the central 80%-confidence interval for the future lifetime according to the AVÖ 2005R.*

The 90%-quantile of the future lifetime increases much less than the expected future lifetime and the 10%-quantile increases much more. The death age will therefore display a smaller variance and the confidence intervals for the remaining lifetime will be smaller.

Interestingly, for 60-year old females the 90%-quantile is even decreased compared to the AVÖ 1996R. This is again a consequence of the higher death probabilities for old ages in the new Austrian population tables, compared to the tables used for the AVÖ 1996R.

The international comparison in Figure 26 draws an ambivalent picture: While the future lifetime develops almost identically to the DAV 2004-R (although the tables were derived from different data with slightly different assumptions as outlined above), the 90%-quantiles of the DAV 2004-R increase faster than the quantiles of the AVÖ 2005R; this again is a consequence of the 1% lower trend bound in the German table. The Swiss values increase faster in all aspects.

5.4.1 Derivation of $\mathring{e}_x(t)$

Lemma 2. *Let $q_x(t)$ be the one-year death probabilities for a person aged $x \in \mathbb{N}_0$ in calendar year t and assume that the force of mortality $\mu_{x+k+u}(t+k+u)$ with $k \in \mathbb{N}_0$ and $u \in [0,1)$ is constant in u. Let $T_x(t)$ denote the future lifetime of a person aged x in calendar year t. Then the expected future lifetime at t can be calculated as*

$$\mathring{e}_x(t) = -\sum_{k=0}^{\infty} \frac{{}_kp_x(t) \cdot q_{x+k}(t+k)}{\log p_{x+k}(t+k)}.$$

Proof. The distribution of $T_x(t)$ is determined using the time-dependent force of mortality $\{\mu_{x+s}(t+s)\}_{s\geq 0}$ as

$$\mathbb{P}(T_x(t) > s) = {}_sp_x(t) = \exp\left(-\int_0^s \mu_{x+u}(t+u)\,du\right), \quad s \geq 0.$$

Note that ${}_{k+u}p_x(t) = {}_kp_x(t)\,{}_up_{x+k}(t+k)$ for $k \in \mathbb{N}_0$ and $u \in [0,1)$. Under the assumption of a yearly constant force of mortality

$$\mu_{x+k+u}(t+k+u) = -\log p_{x+k}(t+k), \quad u \in [0,1),$$

it follows that ${}_up_{x+k}(t+k) = p_{x+k}^u(t+k)$. Due to Fubini's theorem, the expectation can be expressed as

$$\begin{aligned}\mathbb{E}[T_x] &= \int_0^\infty \mathbb{P}(T_x(t) > s)\,ds = \int_0^\infty {}_sp_x(t)\,ds = \sum_{k=0}^{\infty} {}_kp_x(t) \int_0^1 \underbrace{{}_up_{x+k}(t+k)}_{=p_{x+k}^u(t+k)}\,du \\ &= \sum_{k=0}^{\infty} {}_kp_x(t)\, \frac{p_{x+k}^u(t+k)}{\log p_{x+k}(t+k)}\bigg|_{u=0}^{1} = -\sum_{k=0}^{\infty} {}_kp_x(t)\,\frac{q_{x+k}(t+k)}{\log p_{x+k}(t+k)}\end{aligned}$$

□

This result can easily be generalized to non-integer ages $x + u \in \mathbb{R}$. However, the expressions become a bit more complicated.

One also has to notice that under the assumption of a constant force of mortality throughout each year, the one-year survival probabilities $p_{x+k}(t+k)$ cannot vanish (only become very small) as the k-th summand in this case would be singular.

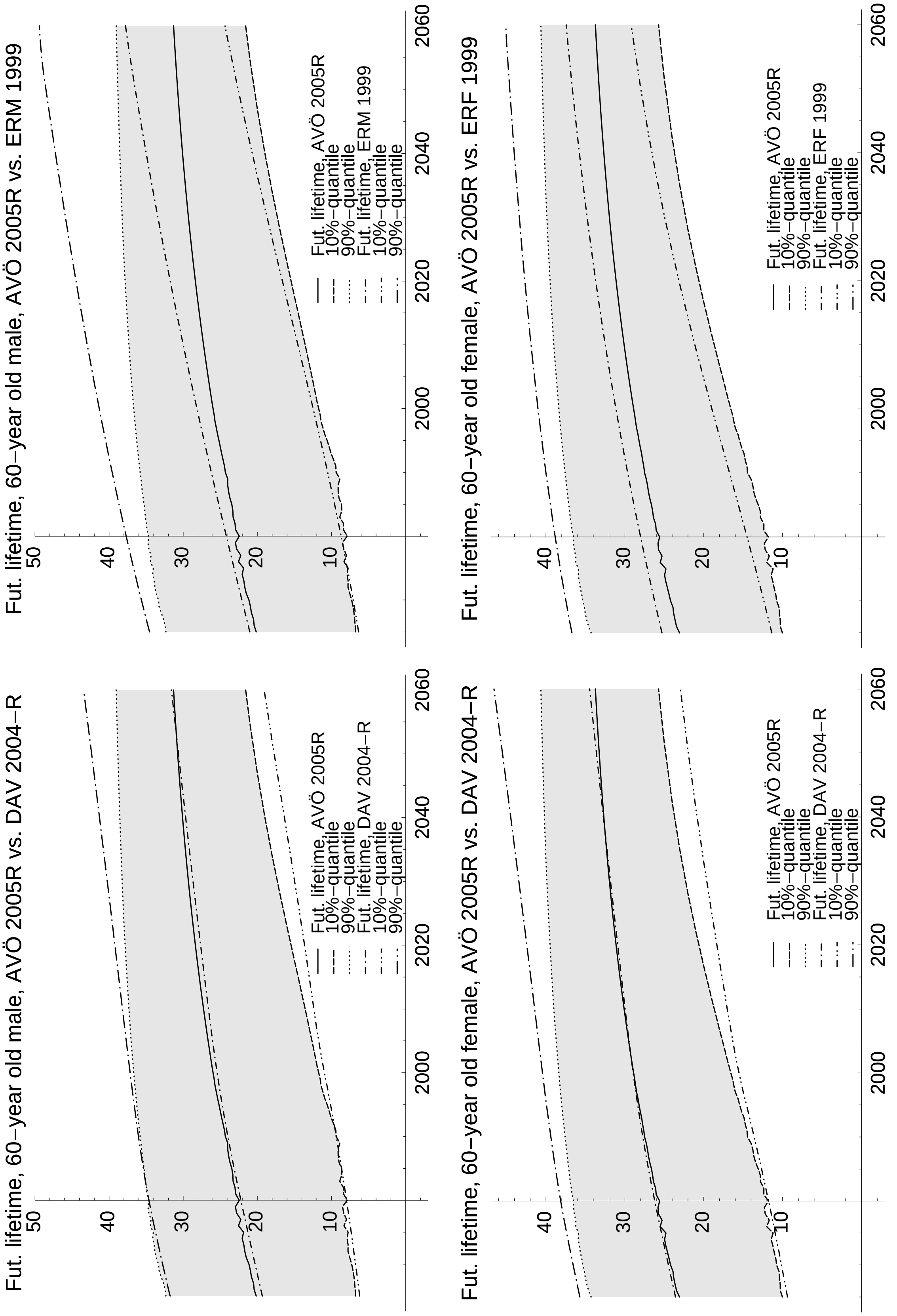


Figure 26: *Expected future lifetime and its 10%- and 90%-quantiles: Comparison of the second-order tables of the AVÖ 2005R with the second-order German and Swiss annuity valuation tables.*

6 Comparison of Net Single and Yearly Premiums

In this chapter we compare actuarial values like net single and yearly premiums of different types of insurance resulting from the new Austrian annuity valuation table AVÖ 2005R to values calculated using the DAV 2004-R, the ERM/F 1999 and the AVÖ 1996R for individual contracts as well as for group contracts. We can see in most of the tabulated results that the Swiss annuity valuation tables ERM/F 1999 are the most expensive ones followed by the German annuity valuation tables DAV 2004-R and the new Austrian annuity valuation tables AVÖ 2005R.

Unless it is explicitly mentioned, the parameters from Table 6 are used. All net premiums and present values are time-dependent and are calculated from the generation life table of a person born in the corresponding birth year. For the AVÖ 2005R we used a maximum age of $\omega = 121$ years for all calculations. However, a different choice does not influence the numerical values significantly.

Parameter	Value
Yearly effective interest rate r	2.25%
Signing year t of the contract	2005
Length n of a temporary insurance	20 years
Premium refund α (if mentioned)	100%

Table 6: *Parameters for the comparisons*

6.1 Net Single Premium of Immediate Life Annuity-Dues

In the following tables we compare the net single premiums

$$\ddot{a}_x(t) = \sum_{k=0}^{\infty} v^k {}_kp_x(t) \quad \text{with } v = \frac{1}{1+r}$$

of an immediate whole life annuity-due of 1 starting in the years $t = 2005$ and 2015 for people aged $x = 20, 25, \ldots, 100$. Although this sum is formally an infinite series, it terminates at a maximum age of 113 for the AVÖ 1996R, at 121 for the DAV 2004-R, and at 140 for the ERM/F 1999. For the AVÖ 2005R we also chose a maximum age of $x = 121$ years for these calculations. As we can see, the differences of the AVÖ 2005R to the AVÖ 1996R are from -14.5% to 18.6% for male persons and from -19.0% to 10.1% for female persons for individual contracts. The largest increases can be observed for the age range from 60 to 75 years, while for ages above 90 years the values even decrease compared to the AVÖ 1996R.

Males

2005		Individual contracts					Group contracts		
Age	Birth	AVÖ05	DAV04	ERM99	AVÖ96	’96→’05	AVÖ05	AVÖ96	’96→’05
20	1985	35.780	36.422	36.435	33.987	5.3%	35.640	33.860	5.3%
25	1980	34.575	35.202	35.343	32.639	5.9%	34.406	32.494	5.9%
30	1975	33.184	33.803	34.121	31.104	6.7%	32.979	30.939	6.6%
35	1970	31.582	32.199	32.696	29.391	7.5%	31.334	29.203	7.3%
40	1965	29.782	30.401	31.038	27.494	8.3%	29.481	27.280	8.1%
45	1960	27.799	28.408	29.134	25.443	9.3%	27.435	25.199	8.9%
50	1955	25.616	26.209	26.947	23.249	10.2%	25.188	22.969	9.7%
55	1950	23.245	23.802	24.502	20.898	11.2%	22.762	20.574	10.6%
60	1945	20.637	21.156	21.833	18.390	12.2%	20.123	18.028	11.6%
65	1940	17.785	18.292	18.979	15.764	12.8%	17.272	15.419	12.0%
70	1935	14.818	15.342	16.022	13.124	12.9%	14.340	12.809	11.9%
75	1930	11.835	12.415	13.083	10.526	12.4%	11.424	10.252	11.4%
80	1925	8.976	9.618	10.317	8.135	10.3%	8.657	7.912	9.4%
85	1920	6.490	7.196	7.879	6.138	5.7%	6.278	5.971	5.1%
90	1915	4.487	5.385	5.878	4.553	−1.5%	4.373	4.440	−1.5%
95	1910	3.021	4.172	4.357	3.323	−9.1%	2.981	3.253	−8.4%
100	1905	2.122	3.371	3.245	2.482	−14.5%	2.122	2.445	−13.2%

2015		Individual contracts					Group contracts		
Age	Birth	AVÖ05	DAV04	ERM99	AVÖ96	’96→’05	AVÖ05	AVÖ96	’96→’05
20	1995	36.098	36.848	36.883	34.204	5.5%	35.975	34.080	5.6%
25	1990	34.940	35.686	35.840	32.872	6.3%	34.793	32.731	6.3%
30	1985	33.610	34.354	34.666	31.359	7.2%	33.434	31.198	7.2%
35	1980	32.087	32.827	33.305	29.669	8.2%	31.874	29.486	8.1%
40	1975	30.375	31.112	31.726	27.795	9.3%	30.117	27.587	9.2%
45	1970	28.479	29.201	29.914	25.762	10.5%	28.168	25.525	10.4%
50	1965	26.382	27.083	27.827	23.579	11.9%	26.017	23.307	11.6%
55	1960	24.088	24.752	25.483	21.232	13.5%	23.673	20.918	13.2%
60	1955	21.552	22.178	22.902	18.722	15.1%	21.107	18.371	14.9%
65	1950	18.762	19.374	20.110	16.085	16.6%	18.312	15.748	16.3%
70	1945	15.818	16.433	17.168	13.421	17.9%	15.391	13.113	17.4%
75	1940	12.800	13.436	14.180	10.790	18.6%	12.424	10.519	18.1%
80	1935	9.824	10.502	11.296	8.356	17.6%	9.525	8.134	17.1%
85	1930	7.137	7.907	8.689	6.308	13.1%	6.932	6.141	12.9%
90	1925	4.910	5.930	6.503	4.673	5.1%	4.798	4.559	5.2%
95	1920	3.253	4.610	4.811	3.400	−4.3%	3.214	3.330	−3.5%
100	1915	2.225	3.744	3.566	2.525	−11.9%	2.225	2.488	−10.5%

Females

2005		Individual contracts					Group contracts		
Age	Birth	AVÖ05	DAV04	ERF99	AVÖ96	’96→’05	AVÖ05	AVÖ96	’96→’05
20	1985	36.683	37.322	37.016	35.773	2.5%	36.611	35.687	2.6%
25	1980	35.556	36.200	35.915	34.552	2.9%	35.470	34.452	3.0%
30	1975	34.266	34.920	34.669	33.170	3.3%	34.161	33.055	3.3%
35	1970	32.803	33.467	33.269	31.622	3.7%	32.675	31.490	3.8%
40	1965	31.157	31.837	31.702	29.899	4.2%	31.000	29.747	4.2%
45	1960	29.323	30.020	29.955	28.004	4.7%	29.133	27.829	4.7%
50	1955	27.281	27.994	28.009	25.929	5.2%	27.055	25.726	5.2%
55	1950	25.014	25.737	25.818	23.651	5.8%	24.755	23.416	5.7%

2005		Individual contracts					Group contracts		
Age	Birth	AVÖ05	DAV04	ERF99	AVÖ96	’96→’05	AVÖ05	AVÖ96	’96→’05
60	1945	22.470	23.227	23.347	21.140	6.3%	22.186	20.874	6.3%
65	1940	19.625	20.467	20.585	18.401	6.7%	19.327	18.131	6.6%
70	1935	16.544	17.488	17.558	15.512	6.7%	16.245	15.247	6.5%
75	1930	13.321	14.406	14.341	12.564	6.0%	13.043	12.317	5.9%
80	1925	10.135	11.330	11.098	9.717	4.3%	9.904	9.502	4.2%
85	1920	7.240	8.526	8.100	7.222	0.3%	7.075	7.051	0.3%
90	1915	4.911	6.406	5.663	5.286	−7.1%	4.819	5.167	−6.7%
95	1910	3.260	4.985	3.952	3.812	−14.5%	3.227	3.737	−13.6%
100	1905	2.258	3.911	2.851	2.788	−19.0%	2.258	2.749	−17.9%

2015		Individual contracts					Group contracts		
Age	Birth	AVÖ05	DAV04	ERF99	AVÖ96	’96→’05	AVÖ05	AVÖ96	’96→’05
20	1995	36.892	37.647	37.237	35.975	2.6%	36.831	35.893	2.6%
25	1990	35.808	36.583	36.176	34.779	3.0%	35.734	34.685	3.0%
30	1985	34.572	35.363	34.978	33.429	3.4%	34.482	33.321	3.5%
35	1980	33.172	33.975	33.632	31.915	3.9%	33.064	31.790	4.0%
40	1975	31.598	32.416	32.124	30.227	4.5%	31.466	30.083	4.6%
45	1970	29.840	30.675	30.441	28.362	5.2%	29.680	28.197	5.3%
50	1965	27.878	28.728	28.563	26.313	5.9%	27.689	26.121	6.0%
55	1960	25.693	26.550	26.449	24.056	6.8%	25.475	23.834	6.9%
60	1955	23.239	24.121	24.061	21.564	7.8%	22.998	21.313	7.9%
65	1950	20.489	21.438	21.382	18.839	8.8%	20.234	18.582	8.9%
70	1945	17.486	18.515	18.423	15.951	9.6%	17.227	15.696	9.8%
75	1940	14.294	15.436	15.233	12.983	10.1%	14.048	12.742	10.2%
80	1935	11.055	12.291	11.940	10.092	9.5%	10.844	9.880	9.8%
85	1930	7.997	9.346	8.789	7.529	6.2%	7.841	7.359	6.6%
90	1925	5.428	7.043	6.128	5.510	−1.5%	5.339	5.390	−1.0%
95	1920	3.564	5.481	4.214	3.959	−10.0%	3.532	3.884	−9.1%
100	1915	2.412	4.326	2.986	2.873	−16.0%	2.412	2.833	−14.9%

6.1.1 Future Trends

In the following tables we compare the evolution of the net single premiums of immediate life annuities of 1 issued to 65-year old male and female persons in the years $t = 2005, 2010, \ldots, 2050$. As we can see, the differences of the AVÖ 2005R to the AVÖ 1996R grow over time from 12.8% to more than 20% for male persons and from 7% to more than 11% for female persons for individual as well as for group contracts.

65-Year Old Males, Immediate Life Annuities

	Individual contracts					Group contracts		
Year	AVÖ05	DAV04	ERM99	AVÖ96	’96→’05	AVÖ05	AVÖ96	’96→’05
2005	17.785	18.292	18.979	15.764	12.8%	17.272	15.419	12.0%
2010	18.292	18.843	19.557	15.928	14.8%	17.811	15.588	14.3%
2015	18.762	19.374	20.110	16.085	16.6%	18.312	15.748	16.3%
2020	19.195	19.887	20.638	16.240	18.2%	18.775	15.907	18.0%
2025	19.593	20.381	21.142	16.393	19.5%	19.200	16.064	19.5%
2030	19.956	20.858	21.623	16.544	20.6%	19.589	16.219	20.8%
2035	20.287	21.319	22.081	16.693	21.5%	19.944	16.372	21.8%
2040	20.588	21.765	22.518	16.840	22.3%	20.267	16.524	22.7%
2045	20.861	22.196	22.935	16.986	22.8%	20.561	16.673	23.3%
2050	21.109	22.614	23.332	17.129	23.2%	20.827	16.821	23.8%

65-Year Old Females, Immediate Life Annuities

	Individual contracts					Group contracts		
Year	AVÖ05	DAV04	ERF99	AVÖ96	'96→'05	AVÖ05	AVÖ96	'96→'05
2005	19.625	20.467	20.585	18.401	6.7%	19.327	18.131	6.6%
2010	20.075	20.961	20.998	18.623	7.8%	19.800	18.360	7.8%
2015	20.489	21.438	21.382	18.839	8.8%	20.234	18.582	8.9%
2020	20.868	21.897	21.740	19.051	9.5%	20.632	18.799	9.8%
2025	21.212	22.339	22.071	19.257	10.2%	20.995	19.012	10.4%
2030	21.526	22.766	22.380	19.459	10.6%	21.325	19.220	11.0%
2035	21.810	23.179	22.667	19.656	11.0%	21.623	19.423	11.3%
2040	22.067	23.577	22.933	19.849	11.2%	21.894	19.621	11.6%
2045	22.300	23.962	23.182	20.037	11.3%	22.139	19.815	11.7%
2050	22.511	24.333	23.414	20.221	11.3%	22.361	20.004	11.8%

6.2 Net Single Premiums of Immediate Temporary Life Annuity-Dues

In the following tables we compare the net single premiums

$$\ddot{a}_{x:\overline{20}|}(t) = \sum_{k=0}^{19} v^k \, {}_kp_x(t)$$

of an immediate 20-year temporary life annuity-due of 1 starting in the years $t = 2005$ and 2015 for people aged $x = 20, 25, \ldots, 90$. As we can see, the differences of the AVÖ 2005R to the AVÖ 1996R are from -1.5% to 16.2% for male persons and from -7.1% to 9.6% for female persons for individual contracts. Again, the largest increases happen for the age range from 70 to 80 years.

Males

2005		Individual contracts					Group contracts		
Age	Birth	AVÖ05	DAV04	ERM99	AVÖ96	'96→'05	AVÖ05	AVÖ96	'96→'05
20	1985	16.237	16.259	16.158	16.232	0.0%	16.237	16.232	0.0%
25	1980	16.233	16.251	16.152	16.224	0.1%	16.233	16.224	0.1%
30	1975	16.212	16.230	16.161	16.187	0.1%	16.211	16.187	0.1%
35	1970	16.157	16.182	16.166	16.118	0.2%	16.153	16.118	0.2%
40	1965	16.072	16.111	16.151	16.003	0.4%	16.059	16.003	0.3%
45	1960	15.965	16.016	16.105	15.846	0.8%	15.931	15.838	0.6%
50	1955	15.818	15.881	15.989	15.622	1.3%	15.750	15.592	1.0%
55	1950	15.608	15.668	15.771	15.265	2.3%	15.489	15.187	2.0%
60	1945	15.230	15.284	15.389	14.667	3.8%	15.047	14.520	3.6%
65	1940	14.516	14.583	14.730	13.680	6.1%	14.259	13.490	5.7%
70	1935	13.299	13.407	13.647	12.208	8.9%	12.970	11.981	8.3%
75	1930	11.394	11.627	12.040	10.249	11.2%	11.035	10.009	10.3%
80	1925	8.921	9.372	9.991	8.087	10.3%	8.609	7.871	9.4%
85	1920	6.488	7.137	7.815	6.134	5.8%	6.276	5.968	5.2%
90	1915	4.486	5.374	5.871	4.553	−1.5%	4.373	4.440	−1.5%

2015		Individual contracts					Group contracts		
Age	Birth	AVÖ05	DAV04	ERM99	AVÖ96	'96→'05	AVÖ05	AVÖ96	'96→'05
20	1995	16.259	16.275	16.190	16.245	0.1%	16.259	16.245	0.1%
25	1990	16.256	16.268	16.186	16.237	0.1%	16.256	16.237	0.1%
30	1985	16.239	16.252	16.194	16.205	0.2%	16.239	16.205	0.2%
35	1980	16.198	16.215	16.200	16.144	0.3%	16.195	16.144	0.3%
40	1975	16.133	16.160	16.193	16.041	0.6%	16.123	16.041	0.5%
45	1970	16.051	16.085	16.163	15.898	1.0%	16.025	15.891	0.8%
50	1965	15.938	15.978	16.078	15.692	1.6%	15.885	15.664	1.4%
55	1960	15.775	15.813	15.914	15.360	2.7%	15.682	15.288	2.6%
60	1955	15.478	15.518	15.622	14.796	4.6%	15.335	14.660	4.6%
65	1950	14.909	14.966	15.098	13.853	7.6%	14.704	13.673	7.5%
70	1945	13.893	13.976	14.193	12.419	11.9%	13.621	12.201	11.6%
75	1940	12.177	12.339	12.759	10.479	16.2%	11.863	10.244	15.8%
80	1935	9.734	10.116	10.798	8.300	17.3%	9.444	8.086	16.8%
85	1930	7.132	7.798	8.575	6.304	13.1%	6.928	6.137	12.9%
90	1925	4.910	5.905	6.487	4.673	5.1%	4.798	4.559	5.2%

Females

2005		Individual contracts					Group contracts		
Age	Birth	AVÖ05	DAV04	ERF99	AVÖ96	'96→'05	AVÖ05	AVÖ96	'96→'05
20	1985	16.296	16.298	16.298	16.296	0.0%	16.296	16.296	0.0%
25	1980	16.289	16.289	16.284	16.288	0.0%	16.289	16.288	0.0%
30	1975	16.270	16.269	16.262	16.267	0.0%	16.270	16.267	0.0%
35	1970	16.237	16.236	16.232	16.231	0.0%	16.235	16.231	0.0%
40	1965	16.190	16.194	16.196	16.177	0.1%	16.184	16.177	0.0%
45	1960	16.134	16.140	16.158	16.110	0.1%	16.120	16.107	0.1%
50	1955	16.063	16.066	16.118	16.021	0.3%	16.035	16.009	0.2%
55	1950	15.960	15.946	16.048	15.869	0.6%	15.912	15.835	0.5%
60	1945	15.748	15.726	15.897	15.560	1.2%	15.672	15.490	1.2%
65	1940	15.284	15.306	15.558	14.932	2.4%	15.166	14.826	2.3%
70	1935	14.343	14.468	14.802	13.775	4.1%	14.168	13.622	4.0%
75	1930	12.599	12.972	13.277	11.938	5.5%	12.374	11.746	5.3%
80	1925	10.024	10.794	10.858	9.579	4.6%	9.802	9.380	4.5%
85	1920	7.234	8.372	8.074	7.207	0.4%	7.070	7.039	0.4%
90	1915	4.911	6.370	5.662	5.285	−7.1%	4.819	5.166	−6.7%

2015		Individual contracts					Group contracts		
Age	Birth	AVÖ05	DAV04	ERF99	AVÖ96	'96→'05	AVÖ05	AVÖ96	'96→'05
20	1995	16.303	16.305	16.303	16.302	0.0%	16.303	16.302	0.0%
25	1990	16.297	16.297	16.292	16.295	0.0%	16.297	16.295	0.0%
30	1985	16.283	16.281	16.274	16.278	0.0%	16.283	16.278	0.0%
35	1980	16.258	16.254	16.250	16.249	0.1%	16.257	16.249	0.1%
40	1975	16.223	16.220	16.222	16.204	0.1%	16.218	16.204	0.1%
45	1970	16.180	16.178	16.193	16.148	0.2%	16.169	16.145	0.2%
50	1965	16.126	16.121	16.164	16.070	0.3%	16.105	16.060	0.3%
55	1960	16.047	16.028	16.116	15.936	0.7%	16.011	15.907	0.7%
60	1955	15.887	15.860	16.009	15.661	1.4%	15.829	15.600	1.5%
65	1950	15.534	15.538	15.760	15.092	2.9%	15.443	14.997	3.0%
70	1945	14.793	14.866	15.173	14.015	5.5%	14.654	13.875	5.6%
75	1940	13.307	13.565	13.879	12.256	8.6%	13.118	12.073	8.7%
80	1935	10.876	11.520	11.604	9.924	9.6%	10.677	9.729	9.7%
85	1930	7.985	9.093	8.749	7.510	6.3%	7.831	7.343	6.6%
90	1925	5.428	6.975	6.126	5.509	−1.5%	5.338	5.389	−0.9%

6.3 Deferred Life Annuities (No Refund of Premiums)

In the following tables we compare the net single premiums ${}_{n|}\ddot{a}_x(t)$ and the net yearly premiums

$$ {}_{n|}P_x(t) = \frac{{}_{n|}\ddot{a}_x(t)}{\ddot{a}_{x:\overline{n}|}(t)} = \frac{{}_nE_x(t)\ddot{a}_{x+n}(t+n)}{\ddot{a}_x(t) - {}_nE_x(t)\ddot{a}_{x+n}(t+n)} $$

of deferred life annuities of 1 (no refund of premiums) issued in year $t = 2005$ to a person aged $x = 20, 25, \dots, 65$ and with an annuitization age of $x + n = 55, 60, 65, 70$ years. The symbol ${}_nE_x(t)$ denotes the net single premium of an n-year term insurance and can thus be calculated as ${}_nE_x(t) = v^n {}_np_x(t)$. As one can see, the differences of the AVÖ 2005R to the AVÖ 1996R are strongly dependent on the length of the accumulation phase, with the largest increases of 18% to 38% for long accumulation phases, while for short accumulation phases of 5 years the increase is only 13% to 18% for males. For females, again the increases are roughly half as large as the male increases.

Males

NSP		Individual contracts					Group contracts		
Start	Age	AVÖ05	DAV04	ERM99	AVÖ96	’96→’05	AVÖ05	AVÖ96	’96→’05
55	20	11.423	12.018	12.225	9.683	18.0%	11.284	9.555	18.1%
	25	12.644	13.236	13.523	10.757	17.5%	12.477	10.612	17.6%
	30	13.976	14.566	14.960	11.946	17.0%	13.774	11.781	16.9%
	35	15.425	16.017	16.531	13.273	16.2%	15.181	13.085	16.0%
	40	17.024	17.620	18.242	14.765	15.3%	16.728	14.552	15.0%
	45	18.819	19.414	20.119	16.482	14.2%	18.460	16.239	13.7%
	50	20.860	21.448	22.177	18.497	12.8%	20.434	18.217	12.2%
60	20	9.294	9.878	10.101	7.592	22.4%	9.158	7.464	22.7%
	25	10.266	10.847	11.143	8.419	21.9%	10.103	8.274	22.1%
	30	11.320	11.898	12.291	9.332	21.3%	11.123	9.167	21.3%
	35	12.461	13.038	13.538	10.349	20.4%	12.223	10.162	20.3%
	40	13.710	14.289	14.887	11.490	19.3%	13.422	11.277	19.0%
	45	15.103	15.681	16.356	12.801	18.0%	14.755	12.557	17.5%
	50	16.675	17.248	17.953	14.335	16.3%	16.260	14.056	15.7%
	55	18.499	19.050	19.742	16.161	14.5%	18.021	15.838	13.8%
65	20	7.406	7.979	8.211	5.760	28.6%	7.276	5.636	29.1%
	25	8.159	8.729	9.027	6.374	28.0%	8.003	6.233	28.4%
	30	8.971	9.536	9.921	7.049	27.3%	8.783	6.888	27.5%
	35	9.842	10.404	10.883	7.798	26.2%	9.616	7.616	26.3%
	40	10.789	11.349	11.916	8.637	24.9%	10.515	8.430	24.7%
	45	11.834	12.391	13.029	9.597	23.3%	11.504	9.361	22.9%
	50	13.002	13.554	14.225	10.719	21.3%	12.609	10.448	20.7%
	55	14.345	14.880	15.552	12.051	19.0%	13.888	11.738	18.3%
	60	15.903	16.417	17.089	13.675	16.3%	15.398	13.321	15.6%
70	20	5.736	6.299	6.533	4.177	37.3%	5.615	4.061	38.3%
	25	6.299	6.858	7.152	4.608	36.7%	6.154	4.477	37.5%
	30	6.900	7.453	7.823	5.081	35.8%	6.726	4.932	36.4%
	35	7.539	8.086	8.539	5.604	34.5%	7.330	5.435	34.9%
	40	8.224	8.768	9.297	6.186	32.9%	7.973	5.995	33.0%
	45	8.973	9.510	10.103	6.852	31.0%	8.671	6.633	30.7%
	50	9.798	10.329	10.958	7.627	28.5%	9.438	7.377	27.9%
	55	10.734	11.251	11.891	8.544	25.6%	10.314	8.257	24.9%
	60	11.804	12.307	12.960	9.660	22.2%	11.334	9.334	21.4%
	65	13.081	13.585	14.264	11.093	17.9%	12.584	10.760	16.9%

Net prem.		Individual contracts					Group contracts		
Start	Age	AVÖ05	DAV04	ERM99	AVÖ96	'96→'05	AVÖ05	AVÖ96	'96→'05
55	20	0.469	0.492	0.505	0.398	17.7%	0.463	0.393	17.9%
	25	0.577	0.603	0.620	0.492	17.3%	0.569	0.485	17.3%
	30	0.728	0.757	0.781	0.624	16.7%	0.717	0.615	16.6%
	35	0.955	0.990	1.023	0.823	15.9%	0.940	0.812	15.8%
	40	1.334	1.379	1.426	1.160	15.0%	1.312	1.143	14.7%
	45	2.096	2.159	2.232	1.839	13.9%	2.057	1.812	13.5%
	50	4.385	4.504	4.650	3.893	12.7%	4.298	3.834	12.1%
60	20	0.351	0.372	0.384	0.288	22.0%	0.346	0.283	22.3%
	25	0.422	0.445	0.460	0.348	21.5%	0.416	0.342	21.7%
	30	0.518	0.543	0.563	0.429	20.8%	0.509	0.421	20.9%
	35	0.652	0.680	0.707	0.544	19.9%	0.640	0.534	19.8%
	40	0.853	0.887	0.922	0.718	18.8%	0.836	0.705	18.6%
	45	1.190	1.232	1.280	1.013	17.5%	1.164	0.993	17.1%
	50	1.865	1.925	1.996	1.608	16.0%	1.821	1.577	15.5%
	55	3.897	4.008	4.147	3.411	14.2%	3.801	3.343	13.7%
65	20	0.261	0.281	0.291	0.204	27.9%	0.257	0.200	28.5%
	25	0.309	0.330	0.343	0.243	27.3%	0.303	0.237	27.7%
	30	0.370	0.393	0.410	0.293	26.4%	0.363	0.286	26.7%
	35	0.453	0.477	0.499	0.361	25.4%	0.443	0.353	25.5%
	40	0.568	0.596	0.623	0.458	24.0%	0.554	0.447	24.0%
	45	0.741	0.774	0.809	0.606	22.4%	0.722	0.591	22.2%
	50	1.031	1.071	1.118	0.856	20.5%	1.002	0.834	20.1%
	55	1.612	1.668	1.738	1.362	18.3%	1.565	1.328	17.8%
	60	3.359	3.464	3.602	2.900	15.8%	3.259	2.830	15.2%
70	20	0.191	0.209	0.218	0.140	36.3%	0.187	0.136	37.2%
	25	0.223	0.242	0.254	0.164	35.5%	0.218	0.160	36.3%
	30	0.263	0.283	0.297	0.195	34.5%	0.256	0.190	35.1%
	35	0.314	0.335	0.353	0.236	33.1%	0.305	0.229	33.5%
	40	0.381	0.405	0.428	0.290	31.4%	0.371	0.282	31.6%
	45	0.477	0.503	0.531	0.369	29.3%	0.462	0.357	29.3%
	50	0.619	0.650	0.685	0.488	26.9%	0.599	0.473	26.7%
	55	0.858	0.896	0.943	0.692	24.0%	0.829	0.670	23.6%
	60	1.336	1.391	1.461	1.107	20.8%	1.290	1.074	20.1%
	65	2.781	2.886	3.025	2.375	17.1%	2.684	2.310	16.2%

Females

NSP		Individual contracts					Group contracts		
Start	Age	AVÖ05	DAV04	ERF99	AVÖ96	'96→'05	AVÖ05	AVÖ96	'96→'05
55	20	12.185	12.823	12.523	11.282	8.0%	12.114	11.195	8.2%
	25	13.511	14.157	13.880	12.514	8.0%	13.425	12.414	8.1%
	30	14.966	15.620	15.379	13.877	7.8%	14.862	13.763	8.0%
	35	16.566	17.230	17.037	15.391	7.6%	16.440	15.259	7.7%
	40	18.334	19.012	18.878	17.081	7.3%	18.180	16.930	7.4%
	45	20.303	20.998	20.933	18.988	6.9%	20.116	18.813	6.9%
	50	22.511	23.223	23.236	21.160	6.4%	22.287	20.957	6.3%
60	20	10.020	10.659	10.359	9.124	9.8%	9.951	9.038	10.1%
	25	11.094	11.741	11.465	10.105	9.8%	11.010	10.005	10.0%
	30	12.268	12.923	12.681	11.187	9.7%	12.166	11.073	9.9%
	35	13.553	14.217	14.022	12.387	9.4%	13.430	12.255	9.6%
	40	14.967	15.644	15.506	13.722	9.1%	14.816	13.570	9.2%
	45	16.534	17.225	17.155	15.225	8.6%	16.350	15.050	8.6%
	50	18.281	18.989	18.996	16.933	8.0%	18.060	16.731	7.9%
	55	20.249	20.971	21.048	18.887	7.2%	19.992	18.652	7.2%

NSP		Individual contracts					Group contracts		
Start	Age	AVÖ05	DAV04	ERF99	AVÖ96	’96→’05	AVÖ05	AVÖ96	’96→’05
65	20	8.092	8.733	8.430	7.207	12.3%	8.025	7.122	12.7%
	25	8.942	9.591	9.311	7.966	12.3%	8.861	7.868	12.6%
	30	9.867	10.525	10.277	8.800	12.1%	9.769	8.687	12.4%
	35	10.875	11.541	11.338	9.722	11.9%	10.756	9.592	12.1%
	40	11.976	12.654	12.506	10.746	11.5%	11.832	10.596	11.7%
	45	13.189	13.880	13.797	11.894	10.9%	13.013	11.722	11.0%
	50	14.532	15.239	15.230	13.195	10.1%	14.320	12.996	10.2%
	55	16.032	16.755	16.817	14.678	9.2%	15.783	14.448	9.2%
	60	17.710	18.469	18.582	16.384	8.1%	17.430	16.122	8.1%
70	20	6.377	7.021	6.710	5.511	15.7%	6.313	5.428	16.3%
	25	7.029	7.682	7.393	6.074	15.7%	6.952	5.979	16.3%
	30	7.735	8.396	8.138	6.692	15.6%	7.642	6.583	16.1%
	35	8.498	9.168	8.951	7.371	15.3%	8.386	7.246	15.7%
	40	9.325	10.006	9.840	8.122	14.8%	9.190	7.979	15.2%
	45	10.229	10.923	10.816	8.961	14.1%	10.064	8.796	14.4%
	50	11.218	11.929	11.891	9.908	13.2%	11.020	9.718	13.4%
	55	12.312	13.041	13.071	10.983	12.1%	12.078	10.764	12.2%
	60	13.520	14.286	14.370	12.215	10.7%	13.253	11.964	10.8%
	65	14.881	15.724	15.830	13.665	8.9%	14.589	13.401	8.9%

Net prem.		Individual contracts					Group contracts		
Start	Age	AVÖ05	DAV04	ERF99	AVÖ96	’96→’05	AVÖ05	AVÖ96	’96→’05
55	20	0.497	0.523	0.511	0.461	8.0%	0.495	0.457	8.2%
	25	0.613	0.642	0.630	0.568	7.9%	0.609	0.563	8.1%
	30	0.775	0.809	0.797	0.719	7.8%	0.770	0.713	7.9%
	35	1.020	1.061	1.050	0.948	7.6%	1.013	0.940	7.7%
	40	1.430	1.482	1.472	1.333	7.3%	1.418	1.321	7.4%
	45	2.251	2.327	2.320	2.106	6.9%	2.231	2.087	6.9%
	50	4.720	4.867	4.869	4.437	6.4%	4.674	4.394	6.3%
60	20	0.376	0.400	0.389	0.342	9.8%	0.373	0.339	10.1%
	25	0.454	0.480	0.469	0.413	9.7%	0.450	0.409	10.0%
	30	0.558	0.588	0.577	0.509	9.6%	0.553	0.504	9.8%
	35	0.704	0.739	0.729	0.644	9.3%	0.698	0.637	9.5%
	40	0.924	0.966	0.957	0.848	9.0%	0.915	0.839	9.1%
	45	1.293	1.346	1.340	1.191	8.5%	1.279	1.178	8.6%
	50	2.031	2.109	2.108	1.882	7.9%	2.008	1.860	8.0%
	55	4.249	4.400	4.413	3.965	7.2%	4.197	3.915	7.2%
65	20	0.283	0.305	0.295	0.252	12.2%	0.281	0.249	12.6%
	25	0.336	0.360	0.350	0.300	12.1%	0.333	0.296	12.5%
	30	0.404	0.431	0.421	0.361	12.0%	0.400	0.357	12.3%
	35	0.496	0.526	0.517	0.444	11.7%	0.491	0.438	12.0%
	40	0.624	0.660	0.651	0.561	11.3%	0.617	0.553	11.6%
	45	0.817	0.860	0.854	0.738	10.7%	0.807	0.728	10.9%
	50	1.140	1.195	1.192	1.036	10.0%	1.124	1.021	10.1%
	55	1.785	1.865	1.868	1.636	9.1%	1.759	1.611	9.2%
	60	3.721	3.882	3.899	3.445	8.0%	3.665	3.392	8.0%
70	20	0.210	0.232	0.221	0.182	15.5%	0.208	0.179	16.1%
	25	0.246	0.269	0.259	0.213	15.5%	0.244	0.210	16.1%
	30	0.292	0.317	0.307	0.253	15.3%	0.288	0.249	15.9%
	35	0.350	0.377	0.368	0.304	15.0%	0.345	0.299	15.5%
	40	0.427	0.458	0.450	0.373	14.5%	0.421	0.367	15.0%
	45	0.536	0.572	0.565	0.471	13.8%	0.528	0.462	14.2%
	50	0.698	0.742	0.738	0.618	12.9%	0.687	0.607	13.2%
	55	0.969	1.027	1.025	0.867	11.8%	0.953	0.851	12.0%
	60	1.511	1.598	1.601	1.369	10.4%	1.484	1.343	10.5%
	65	3.136	3.315	3.329	2.886	8.7%	3.079	2.833	8.7%

6.4 Deferred Life Annuities (Refund of Net Premiums)

In the following tables we compare the net yearly premiums

$$_{n|}P_x^{\mathrm{ref}}(t) = \frac{_{n|}\ddot{a}_x(t)}{\ddot{a}_{x:\overline{n}|}(t) - \alpha\,(IA)^{1}_{x:\overline{n}|}(t)} = \frac{_nE_x(t)\ddot{a}_{x+n}(t+n)}{\ddot{a}_x(t) - {_nE_x(t)}\ddot{a}_{x+n}(t+n) - \alpha\,(IA)^{1}_{x:\overline{n}|}(t)}$$

of deferred life annuities of 1 issued in $t = 2005$ to a person aged $x = 20, 25, \ldots, 65$ and with an annuitization age of $x + n = 55, 60, 65, 70$ years. If the insured dies before the end of the accumulation phase, a fraction α of the accumulated net premium payments so far is paid back (without interest) at the end of the year of death. This refund of the accumulated net premiums can be modelled by a standard increasing n-year term insurance

$$(IA)^{1}_{x:\overline{n}|}(t) = \sum_{k=0}^{n-1}(k+1)v^{k+1}{_kp_x(t)} \cdot q_{x+k}(t+k)$$

of an amount $\alpha\,{_{n|}P_x^{\mathrm{ref}}(t)}$. In our examples, the extreme case $\alpha = 1$ is used. As one can see, the differences of the AVÖ 2005R to the AVÖ 1996R are similar to the case without refund, but the relative changes are a little smaller than without refund.

Males

Net prem.		Individual contracts					Group contracts		
Start	Age	AVÖ05	DAV04	ERM99	AVÖ96	’96→’05	AVÖ05	AVÖ96	’96→’05
55	20	0.475	0.498	0.511	0.407	16.7%	0.470	0.402	16.9%
	25	0.585	0.610	0.627	0.503	16.3%	0.578	0.496	16.4%
	30	0.739	0.767	0.789	0.638	15.8%	0.729	0.629	15.8%
	35	0.970	1.003	1.033	0.842	15.2%	0.956	0.830	15.1%
	40	1.355	1.397	1.438	1.185	14.4%	1.334	1.167	14.2%
	45	2.124	2.183	2.249	1.872	13.5%	2.087	1.844	13.2%
	50	4.427	4.540	4.675	3.937	12.4%	4.343	3.878	12.0%
60	20	0.357	0.378	0.389	0.297	20.3%	0.352	0.292	20.7%
	25	0.430	0.453	0.467	0.359	19.8%	0.424	0.353	20.1%
	30	0.529	0.553	0.571	0.443	19.2%	0.521	0.436	19.4%
	35	0.666	0.694	0.716	0.563	18.4%	0.655	0.553	18.6%
	40	0.872	0.904	0.934	0.742	17.5%	0.857	0.729	17.5%
	45	1.215	1.255	1.297	1.044	16.4%	1.192	1.025	16.3%
	50	1.900	1.955	2.020	1.649	15.2%	1.860	1.618	14.9%
	55	3.946	4.051	4.183	3.467	13.8%	3.856	3.401	13.4%
65	20	0.267	0.286	0.296	0.213	25.0%	0.263	0.209	25.6%
	25	0.316	0.337	0.349	0.255	24.3%	0.311	0.250	24.8%
	30	0.381	0.403	0.418	0.308	23.5%	0.374	0.302	23.9%
	35	0.466	0.490	0.509	0.380	22.5%	0.458	0.373	22.8%
	40	0.586	0.613	0.636	0.483	21.4%	0.574	0.473	21.5%
	45	0.765	0.796	0.826	0.637	20.0%	0.749	0.624	20.0%
	50	1.062	1.100	1.143	0.896	18.5%	1.038	0.877	18.3%
	55	1.653	1.707	1.772	1.415	16.8%	1.612	1.385	16.4%
	60	3.417	3.520	3.654	2.974	14.9%	3.327	2.910	14.3%
70	20	0.197	0.215	0.223	0.150	31.2%	0.193	0.146	32.1%
	25	0.230	0.249	0.260	0.177	30.3%	0.226	0.172	31.0%
	30	0.272	0.292	0.305	0.211	29.2%	0.267	0.206	29.8%
	35	0.326	0.348	0.364	0.255	27.9%	0.320	0.249	28.3%
	40	0.398	0.422	0.441	0.315	26.4%	0.390	0.308	26.7%

Net prem.		Individual contracts					Group contracts		
Start	Age	AVÖ05	DAV04	ERM99	AVÖ96	’96→’05	AVÖ05	AVÖ96	’96→’05
	45	0.499	0.525	0.549	0.400	24.6%	0.488	0.391	24.7%
	50	0.649	0.680	0.711	0.529	22.7%	0.633	0.516	22.6%
	55	0.898	0.936	0.978	0.745	20.5%	0.874	0.727	20.2%
	60	1.391	1.445	1.511	1.178	18.1%	1.353	1.150	17.6%
	65	2.863	2.968	3.101	2.477	15.6%	2.777	2.418	14.9%

Females

Net prem.		Individual contracts					Group contracts		
Start	Age	AVÖ05	DAV04	ERF99	AVÖ96	’96→’05	AVÖ05	AVÖ96	’96→’05
55	20	0.501	0.527	0.515	0.464	7.8%	0.498	0.461	8.0%
	25	0.617	0.647	0.635	0.573	7.8%	0.614	0.568	8.0%
	30	0.782	0.816	0.804	0.726	7.6%	0.777	0.720	7.8%
	35	1.029	1.070	1.058	0.957	7.4%	1.022	0.949	7.6%
	40	1.441	1.494	1.483	1.345	7.2%	1.430	1.333	7.3%
	45	2.267	2.343	2.334	2.123	6.8%	2.248	2.103	6.9%
	50	4.743	4.889	4.888	4.460	6.3%	4.698	4.417	6.4%
60	20	0.379	0.403	0.392	0.346	9.5%	0.377	0.343	9.8%
	25	0.458	0.485	0.473	0.419	9.4%	0.455	0.414	9.7%
	30	0.564	0.594	0.582	0.516	9.3%	0.559	0.511	9.6%
	35	0.712	0.747	0.736	0.653	9.1%	0.706	0.646	9.3%
	40	0.935	0.977	0.967	0.860	8.7%	0.927	0.851	9.0%
	45	1.307	1.361	1.352	1.207	8.3%	1.295	1.193	8.5%
	50	2.050	2.128	2.123	1.902	7.8%	2.029	1.880	7.9%
	55	4.275	4.427	4.434	3.992	7.1%	4.226	3.944	7.2%
65	20	0.286	0.309	0.298	0.256	11.6%	0.284	0.253	12.1%
	25	0.340	0.365	0.354	0.305	11.6%	0.337	0.301	12.0%
	30	0.410	0.438	0.426	0.368	11.4%	0.406	0.364	11.8%
	35	0.503	0.535	0.523	0.453	11.2%	0.499	0.447	11.5%
	40	0.634	0.671	0.660	0.573	10.8%	0.628	0.565	11.1%
	45	0.830	0.874	0.865	0.753	10.3%	0.822	0.743	10.5%
	50	1.157	1.213	1.205	1.055	9.6%	1.143	1.041	9.8%
	55	1.807	1.890	1.886	1.661	8.8%	1.784	1.638	8.9%
	60	3.752	3.918	3.925	3.480	7.8%	3.700	3.431	7.9%
70	20	0.213	0.235	0.224	0.186	14.5%	0.212	0.184	15.1%
	25	0.250	0.274	0.263	0.219	14.4%	0.248	0.216	15.0%
	30	0.297	0.323	0.311	0.260	14.2%	0.294	0.256	14.8%
	35	0.357	0.385	0.374	0.313	13.9%	0.353	0.309	14.4%
	40	0.437	0.469	0.458	0.385	13.4%	0.432	0.379	13.9%
	45	0.548	0.586	0.575	0.486	12.8%	0.542	0.479	13.1%
	50	0.715	0.761	0.750	0.639	11.9%	0.706	0.629	12.2%
	55	0.991	1.052	1.042	0.894	10.9%	0.977	0.879	11.1%
	60	1.541	1.632	1.624	1.405	9.7%	1.518	1.382	9.8%
	65	3.182	3.365	3.365	2.939	8.3%	3.130	2.890	8.3%

6.5 Deferred Life Annuities (Refund of Net Premiums and Guarantee Period)

In the following tables we compare the net yearly premiums

$$ {}_{n|}P_x^{\mathrm{ref},m}(t) = \frac{{}_nE_x(t)\cdot \ddot{a}_{\overline{m}|}(t) + {}_{n+m}E_x(t)\cdot \ddot{a}_{x+n+m}(t+n+m)}{\ddot{a}_{x:\overline{n}|}(t) - \alpha\,(IA)^1_{x:\overline{n}|}(t)} $$

of deferred life annuities of 1 with m years of guaranteed payments, issued in $t = 2005$ to a person aged $x = 20, 25, \ldots, 65$ and with an annuitization age of $x + n = 55, 60, 65, 70$ years. If the insured dies before the end of the accumulation phase, a fraction α of the accumulated net premium payments so far is paid back (without interest) at the end of the year of death. In our examples, the extreme case $\alpha = 1$ is used. After annuitization has started, payments are guaranteed for a period of m years regardless of the possible death of the insured. In the examples, $m = 15$ is used. Some Austrian annuity contracts offer both features, a premium refund as well as a certain guarantee period, so these comparisons can give insurance companies the best estimate of the changes compared to the previous valuation table.

As the mortality reduction is irrelevant for the payments during the guaranteed period, the relative differences of the AVÖ 2005R to the AVÖ 1996R are considerably smaller than for the case without such a guaranteed payment period.

Males

Net prem.		Individual contracts					Group contracts		
Start	Age	AVÖ05	DAV04	ERM99	AVÖ96	'96→'05	AVÖ05	AVÖ96	'96→'05
55	20	0.478	0.501	0.513	0.414	15.4%	0.473	0.409	15.6%
	25	0.589	0.614	0.630	0.512	15.0%	0.582	0.506	15.1%
	30	0.745	0.773	0.793	0.650	14.5%	0.736	0.642	14.5%
	35	0.979	1.011	1.039	0.860	13.9%	0.966	0.849	13.8%
	40	1.369	1.410	1.448	1.210	13.1%	1.350	1.195	13.0%
	45	2.150	2.207	2.267	1.915	12.3%	2.118	1.890	12.0%
	50	4.490	4.597	4.720	4.033	11.3%	4.416	3.980	11.0%
60	20	0.360	0.381	0.391	0.306	17.8%	0.356	0.302	18.0%
	25	0.435	0.457	0.470	0.371	17.3%	0.429	0.366	17.4%
	30	0.535	0.559	0.575	0.458	16.6%	0.528	0.452	16.7%
	35	0.675	0.702	0.723	0.582	15.9%	0.666	0.574	15.9%
	40	0.886	0.917	0.945	0.770	15.0%	0.872	0.759	14.9%
	45	1.237	1.275	1.314	1.085	14.0%	1.217	1.070	13.8%
	50	1.938	1.992	2.051	1.717	12.9%	1.905	1.693	12.6%
	55	4.039	4.140	4.260	3.618	11.6%	3.966	3.566	11.2%
65	20	0.271	0.290	0.299	0.226	20.1%	0.268	0.222	20.3%
	25	0.322	0.342	0.353	0.270	19.4%	0.318	0.266	19.5%
	30	0.388	0.409	0.423	0.327	18.5%	0.383	0.323	18.6%
	35	0.476	0.500	0.517	0.405	17.6%	0.469	0.399	17.6%
	40	0.600	0.626	0.648	0.515	16.5%	0.591	0.508	16.4%
	45	0.787	0.817	0.845	0.682	15.3%	0.774	0.673	15.1%
	50	1.097	1.134	1.172	0.963	14.0%	1.079	0.949	13.6%
	55	1.716	1.768	1.827	1.525	12.5%	1.686	1.504	12.1%
	60	3.571	3.669	3.787	3.218	11.0%	3.506	3.174	10.4%
70	20	0.202	0.219	0.227	0.166	21.4%	0.199	0.164	21.7%
	25	0.237	0.255	0.265	0.197	20.5%	0.234	0.194	20.6%
	30	0.281	0.301	0.313	0.236	19.4%	0.278	0.232	19.4%
	35	0.339	0.359	0.374	0.287	18.2%	0.334	0.283	18.1%
	40	0.416	0.438	0.456	0.356	16.8%	0.410	0.351	16.7%
	45	0.524	0.549	0.571	0.454	15.4%	0.516	0.448	15.1%
	50	0.687	0.715	0.743	0.603	13.8%	0.676	0.596	13.5%
	55	0.958	0.993	1.031	0.854	12.2%	0.942	0.843	11.7%
	60	1.500	1.550	1.607	1.357	10.5%	1.475	1.342	10.0%
	65	3.123	3.218	3.332	2.871	8.8%	3.072	2.839	8.2%

Females

Net prem.		Individual contracts					Group contracts		
Start	Age	AVÖ05	DAV04	ERF99	AVÖ96	'96→'05	AVÖ05	AVÖ96	'96→'05
55	20	0.502	0.529	0.516	0.467	7.6%	0.500	0.463	7.8%
	25	0.619	0.649	0.636	0.576	7.5%	0.616	0.572	7.7%
	30	0.785	0.819	0.806	0.731	7.4%	0.780	0.725	7.6%
	35	1.033	1.075	1.061	0.964	7.2%	1.026	0.956	7.4%
	40	1.449	1.502	1.488	1.355	6.9%	1.438	1.344	7.1%
	45	2.280	2.357	2.344	2.140	6.6%	2.262	2.121	6.7%
	50	4.774	4.922	4.912	4.499	6.1%	4.734	4.459	6.2%
60	20	0.381	0.405	0.393	0.349	9.0%	0.378	0.346	9.3%
	25	0.460	0.487	0.475	0.423	8.9%	0.457	0.419	9.2%
	30	0.567	0.597	0.584	0.521	8.7%	0.563	0.516	9.0%
	35	0.716	0.752	0.739	0.660	8.5%	0.711	0.654	8.7%
	40	0.942	0.985	0.972	0.871	8.2%	0.935	0.862	8.4%
	45	1.318	1.373	1.360	1.223	7.8%	1.307	1.211	7.9%
	50	2.069	2.149	2.137	1.930	7.2%	2.051	1.911	7.3%
	55	4.322	4.478	4.469	4.055	6.6%	4.280	4.013	6.6%
65	20	0.288	0.311	0.299	0.261	10.5%	0.286	0.258	10.8%
	25	0.343	0.368	0.355	0.311	10.3%	0.340	0.308	10.7%
	30	0.414	0.442	0.429	0.376	10.1%	0.411	0.372	10.4%
	35	0.508	0.540	0.527	0.463	9.8%	0.505	0.458	10.1%
	40	0.642	0.678	0.665	0.586	9.4%	0.636	0.580	9.7%
	45	0.842	0.886	0.872	0.773	8.9%	0.834	0.765	9.1%
	50	1.175	1.232	1.218	1.085	8.3%	1.164	1.073	8.4%
	55	1.841	1.925	1.910	1.712	7.5%	1.822	1.693	7.6%
	60	3.834	4.001	3.984	3.595	6.6%	3.793	3.556	6.7%
70	20	0.216	0.238	0.226	0.194	11.7%	0.215	0.192	12.1%
	25	0.254	0.278	0.265	0.228	11.5%	0.252	0.226	11.9%
	30	0.302	0.327	0.314	0.272	11.2%	0.300	0.269	11.5%
	35	0.364	0.392	0.378	0.328	10.8%	0.361	0.325	11.1%
	40	0.446	0.478	0.464	0.405	10.2%	0.443	0.400	10.5%
	45	0.562	0.600	0.585	0.513	9.6%	0.557	0.507	9.8%
	50	0.736	0.782	0.765	0.676	8.8%	0.729	0.669	9.0%
	55	1.026	1.086	1.067	0.951	7.9%	1.016	0.940	8.0%
	60	1.605	1.693	1.670	1.501	6.9%	1.588	1.485	7.0%
	65	3.338	3.514	3.479	3.155	5.8%	3.302	3.122	5.8%

7 Methodic Changes Compared to the AVÖ 1996R

The following table shows all changes compared to the AVÖ 1996R at a quick glance, together with a classification as first- or second-order effects:

Effect	2nd O (real)	1st O (secure)
No long-term trend decline (Section 4.6.2)	+	
Corrected selection factors (Section 4.3.1)	+	
Higher future selection for females (Section 4.3.3)	(+)	+
No security margin on the selection[17]		−
0.2% additive selection effect on trend (Section 4.6.1)[18]	+	
0.3% additive security margin on trend (Section 4.7)		+
Increase of the hump in the trend for old ages (Section 4.7.2)	+	+
Linearization of the trend to ensure monotone q_x (Section 4.7.3)		(+)[19]

8 Acknowledgments

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[17] The DAV 2004-R table uses a 10% security margin on the base table to account for model and parameter risk.

[18] Accounts only for the selection due to social status, not due to personal health.

[19] In real-life calculations, this modification has no effect, as it mostly happens in the deferral time.

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A The AVÖ 2005R at a Quick Glance

A.1 The General Formula for the Exact Table

Formula for the death probability of an annuitant of age x in calendar year $t \geq 2001$:

$$q_x(t) = q_x^{\text{base}}(2001)e^{-G(t)\lambda_x} \tag{16}$$

with

$$G(t) = t_{1/2} \arctan\left(\frac{t-2001}{t_{1/2}}\right)$$

and $t_{1/2} = 100$. The base tables $q_x^{\text{base}}(2001)$ and the trends λ_x are tabulated in the following subsections. The tables for individual contracts and group contracts employ the same trend, but use a different base table.

A.2 The Values $q_x^{\text{base}}(2001)$ of the Base Table 2001

As described in Section 4.3.3, the first-order female base table contains a security margin, while for males the first- and second-order base tables coincide.

	Individual contracts			Group contracts		Selection factors			
Age	q_x Males	q_y Fem.	q_y 2.O.	q_x Males	q_y Fem.	M	F	M Gr	F Gr
0	0.004274	0.003009	0.003197	0.004274	0.003009	0.800	0.800	0.800	0.800
1	0.000276	0.000261	0.000278	0.000276	0.000261	0.800	0.800	0.800	0.800
2	0.000211	0.000169	0.000179	0.000211	0.000169	0.800	0.800	0.800	0.800
3	0.000159	0.000105	0.000112	0.000159	0.000105	0.800	0.800	0.800	0.800
4	0.000121	0.000080	0.000085	0.000121	0.000080	0.800	0.800	0.800	0.800
5	0.000104	0.000069	0.000073	0.000104	0.000069	0.800	0.800	0.800	0.800
6	0.000098	0.000068	0.000072	0.000098	0.000068	0.800	0.800	0.800	0.800
7	0.000094	0.000072	0.000077	0.000094	0.000072	0.800	0.800	0.800	0.800
8	0.000088	0.000074	0.000079	0.000088	0.000074	0.800	0.800	0.800	0.800
9	0.000089	0.000075	0.000079	0.000089	0.000075	0.800	0.800	0.800	0.800
10	0.000091	0.000077	0.000082	0.000091	0.000077	0.800	0.800	0.800	0.800
11	0.000090	0.000081	0.000086	0.000090	0.000081	0.800	0.800	0.800	0.800
12	0.000092	0.000088	0.000093	0.000092	0.000088	0.800	0.800	0.800	0.800
13	0.000114	0.000103	0.000110	0.000114	0.000103	0.800	0.800	0.800	0.800
14	0.000177	0.000134	0.000142	0.000177	0.000134	0.800	0.800	0.800	0.800
15	0.000294	0.000167	0.000177	0.000294	0.000167	0.800	0.800	0.800	0.800
16	0.000450	0.000199	0.000211	0.000450	0.000199	0.800	0.800	0.800	0.800
17	0.000620	0.000239	0.000254	0.000620	0.000239	0.800	0.800	0.800	0.800
18	0.000758	0.000268	0.000284	0.000758	0.000268	0.800	0.800	0.800	0.800
19	0.000817	0.000270	0.000287	0.000817	0.000270	0.800	0.800	0.800	0.800
20	0.000821	0.000259	0.000275	0.000821	0.000259	0.800	0.800	0.800	0.800
21	0.000819	0.000247	0.000262	0.000819	0.000247	0.800	0.800	0.800	0.800
22	0.000813	0.000238	0.000253	0.000813	0.000238	0.800	0.800	0.800	0.800
23	0.000805	0.000226	0.000241	0.000805	0.000226	0.800	0.800	0.800	0.800
24	0.000800	0.000216	0.000229	0.000800	0.000216	0.800	0.800	0.800	0.800
25	0.000798	0.000216	0.000229	0.000798	0.000216	0.800	0.800	0.800	0.800
26	0.000788	0.000224	0.000238	0.000788	0.000224	0.800	0.800	0.800	0.800
27	0.000770	0.000232	0.000247	0.000770	0.000232	0.800	0.800	0.800	0.800
28	0.000743	0.000240	0.000255	0.000743	0.000240	0.800	0.800	0.800	0.800
29	0.000717	0.000247	0.000263	0.000717	0.000247	0.800	0.800	0.800	0.800
30	0.000703	0.000258	0.000274	0.000703	0.000258	0.800	0.800	0.800	0.800
31	0.000712	0.000282	0.000300	0.000712	0.000282	0.800	0.800	0.800	0.800

Age	Individual contracts			Group contracts		Selection factors			
	q_x Males	q_y Fem.	q_y 2.O.	q_x Males	q_y Fem.	M	F	M Gr	F Gr
32	0.000747	0.000317	0.000337	0.000747	0.000317	0.800	0.800	0.800	0.800
33	0.000793	0.000361	0.000383	0.000793	0.000361	0.800	0.800	0.800	0.800
34	0.000842	0.000400	0.000425	0.000842	0.000400	0.800	0.800	0.800	0.800
35	0.000893	0.000432	0.000458	0.000893	0.000432	0.800	0.800	0.800	0.800
36	0.000958	0.000468	0.000497	0.000958	0.000468	0.800	0.800	0.800	0.800
37	0.001053	0.000523	0.000555	0.001053	0.000523	0.800	0.800	0.800	0.800
38	0.001176	0.000593	0.000630	0.001176	0.000593	0.800	0.800	0.800	0.800
39	0.001318	0.000673	0.000715	0.001318	0.000673	0.800	0.800	0.800	0.800
40	0.001472	0.000760	0.000808	0.001472	0.000760	0.800	0.800	0.800	0.800
41	0.001601	0.000837	0.000891	0.001611	0.000842	0.786	0.788	0.791	0.792
42	0.001725	0.000914	0.000973	0.001748	0.000923	0.771	0.775	0.781	0.783
43	0.001852	0.000991	0.001056	0.001889	0.001007	0.757	0.763	0.772	0.775
44	0.001991	0.001069	0.001141	0.002044	0.001093	0.742	0.750	0.762	0.767
45	0.002146	0.001154	0.001232	0.002220	0.001186	0.728	0.738	0.753	0.758
46	0.002321	0.001255	0.001341	0.002419	0.001297	0.713	0.725	0.743	0.750
47	0.002512	0.001368	0.001465	0.002638	0.001424	0.698	0.712	0.734	0.741
48	0.002724	0.001487	0.001593	0.002883	0.001557	0.684	0.700	0.724	0.733
49	0.002961	0.001606	0.001723	0.003160	0.001693	0.670	0.688	0.714	0.725
50	0.003221	0.001732	0.001860	0.003466	0.001838	0.655	0.675	0.705	0.716
51	0.003495	0.001860	0.002000	0.003795	0.001987	0.641	0.663	0.696	0.708
52	0.003769	0.001985	0.002137	0.004130	0.002136	0.626	0.650	0.686	0.700
53	0.004034	0.002101	0.002266	0.004463	0.002278	0.612	0.638	0.676	0.691
54	0.004285	0.002209	0.002386	0.004787	0.002414	0.597	0.625	0.667	0.683
55	0.004521	0.002313	0.002502	0.005103	0.002547	0.582	0.612	0.658	0.674
56	0.004747	0.002415	0.002616	0.005415	0.002680	0.568	0.600	0.648	0.666
57	0.004967	0.002511	0.002725	0.005730	0.002811	0.554	0.588	0.638	0.658
58	0.005187	0.002605	0.002831	0.006053	0.002941	0.539	0.575	0.629	0.649
59	0.005416	0.002707	0.002947	0.006397	0.003084	0.524	0.562	0.619	0.641
60	0.005669	0.002828	0.003085	0.006780	0.003253	0.510	0.550	0.610	0.632
61	0.006140	0.003045	0.003321	0.007342	0.003501	0.510	0.550	0.610	0.633
62	0.006700	0.003305	0.003604	0.008008	0.003798	0.511	0.551	0.611	0.633
63	0.007367	0.003621	0.003947	0.008796	0.004159	0.513	0.553	0.612	0.635
64	0.008146	0.004000	0.004357	0.009713	0.004589	0.515	0.554	0.614	0.636
65	0.009033	0.004446	0.004839	0.010751	0.005094	0.518	0.557	0.616	0.638
66	0.010028	0.004973	0.005407	0.011910	0.005689	0.521	0.560	0.619	0.641
67	0.011137	0.005588	0.006069	0.013194	0.006381	0.525	0.564	0.622	0.644
68	0.012373	0.006296	0.006828	0.014616	0.007173	0.530	0.568	0.626	0.647
69	0.013747	0.007106	0.007695	0.016187	0.008078	0.535	0.573	0.630	0.651
70	0.015275	0.008044	0.008697	0.017923	0.009121	0.541	0.578	0.634	0.655
71	0.016979	0.009134	0.009857	0.019847	0.010326	0.547	0.584	0.639	0.660
72	0.018887	0.010407	0.011208	0.021989	0.011730	0.554	0.590	0.645	0.666
73	0.021041	0.011898	0.012788	0.024391	0.013367	0.562	0.598	0.651	0.671
74	0.023487	0.013649	0.014639	0.027103	0.015282	0.570	0.605	0.658	0.678
75	0.026281	0.015704	0.016804	0.030183	0.017519	0.579	0.613	0.665	0.684
76	0.029487	0.018107	0.019329	0.033696	0.020124	0.588	0.622	0.672	0.691
77	0.033173	0.020910	0.022267	0.037714	0.023149	0.599	0.631	0.680	0.699
78	0.037423	0.024176	0.025679	0.042322	0.026657	0.609	0.641	0.689	0.707
79	0.042336	0.027979	0.029642	0.047619	0.030723	0.621	0.652	0.698	0.715
80	0.048029	0.032405	0.034239	0.053724	0.035431	0.633	0.663	0.708	0.724
81	0.054635	0.037550	0.039568	0.060770	0.040879	0.645	0.674	0.717	0.734
82	0.062310	0.043530	0.045742	0.068913	0.047180	0.658	0.686	0.728	0.744
83	0.070771	0.050479	0.052897	0.077821	0.054468	0.672	0.699	0.739	0.754
84	0.079759	0.058554	0.061185	0.087195	0.062896	0.686	0.712	0.750	0.765
85	0.089559	0.067933	0.070785	0.097339	0.072639	0.701	0.726	0.762	0.776
86	0.100315	0.078826	0.081902	0.108394	0.083901	0.717	0.740	0.775	0.788
87	0.112297	0.090922	0.094200	0.120634	0.096330	0.733	0.755	0.788	0.800

Age	Individual contracts			Group contracts		Selection factors			
	q_x Males	q_y Fem.	q_y 2.O.	q_x Males	q_y Fem.	M	F	M Gr	F Gr
88	0.125780	0.104258	0.107709	0.134331	0.109952	0.750	0.770	0.801	0.813
89	0.141175	0.119350	0.122949	0.149900	0.125288	0.768	0.787	0.815	0.826
90	0.158740	0.136380	0.140095	0.167580	0.142509	0.786	0.803	0.829	0.839
91	0.178703	0.155445	0.159229	0.187576	0.161689	0.804	0.820	0.844	0.853
92	0.200998	0.176711	0.180507	0.209784	0.182974	0.824	0.838	0.860	0.868
93	0.225437	0.200275	0.204010	0.233973	0.206437	0.844	0.856	0.875	0.883
94	0.251801	0.226048	0.229632	0.259888	0.231961	0.864	0.875	0.892	0.898
95	0.279916	0.253764	0.257088	0.287327	0.259249	0.885	0.895	0.909	0.914
96	0.309689	0.283071	0.286012	0.316177	0.287923	0.907	0.915	0.926	0.930
97	0.341090	0.313971	0.316395	0.346390	0.317970	0.929	0.935	0.944	0.947
98	0.374103	0.346524	0.348291	0.377934	0.349439	0.952	0.956	0.962	0.964
99	0.408723	0.380731	0.381692	0.410791	0.382317	0.976	0.978	0.981	0.982
100	0.444942	0.416560	0.416560	0.444942	0.416560	1.000	1.000	1.000	1.000
101	0.471086	0.443852	0.443852	0.471086	0.443852	1.000	1.000	1.000	1.000
102	0.497257	0.471177	0.471177	0.497257	0.471177	1.000	1.000	1.000	1.000
103	0.523434	0.498504	0.498504	0.523434	0.498504	1.000	1.000	1.000	1.000
104	0.549612	0.525830	0.525830	0.549612	0.525830	1.000	1.000	1.000	1.000
105	0.575790	0.553159	0.553159	0.575790	0.553159	1.000	1.000	1.000	1.000
106	0.601969	0.580489	0.580489	0.601969	0.580489	1.000	1.000	1.000	1.000
107	0.628148	0.607819	0.607819	0.628148	0.607819	1.000	1.000	1.000	1.000
108	0.654329	0.635150	0.635150	0.654329	0.635150	1.000	1.000	1.000	1.000
109	0.680513	0.662480	0.662480	0.680513	0.662480	1.000	1.000	1.000	1.000
110	0.706699	0.689810	0.689810	0.706699	0.689810	1.000	1.000	1.000	1.000
111	0.732889	0.717139	0.717139	0.732889	0.717139	1.000	1.000	1.000	1.000
112	0.759082	0.744468	0.744468	0.759082	0.744468	1.000	1.000	1.000	1.000
113	0.787559	0.777845	0.777845	0.787559	0.777845	1.000	1.000	1.000	1.000
114	0.810683	0.803081	0.803081	0.810683	0.803081	1.000	1.000	1.000	1.000
115	0.832543	0.826926	0.826926	0.832543	0.826926	1.000	1.000	1.000	1.000
116	0.853040	0.849248	0.849248	0.853040	0.849248	1.000	1.000	1.000	1.000
117	0.872094	0.869938	0.869938	0.872094	0.869938	1.000	1.000	1.000	1.000
118	0.889648	0.888919	0.888919	0.889648	0.888919	1.000	1.000	1.000	1.000
119	0.905666	0.906141	0.906141	0.905666	0.906141	1.000	1.000	1.000	1.000
120	0.920139	0.921588	0.921588	0.920139	0.921588	1.000	1.000	1.000	1.000

If values for ages above $x = 120$ years are required, they can easily be obtained from Equation (3). For all practical calculations, we recommend to introduce a maximum age of 120 or 121 years.

A.3 The Values λ_x of the Initial Trends

	Initial trend, 1st Order		Initial trend, 2nd Order		Limiting probabilities	
Age	λ_x Males	λ_y Fem.	$\lambda_x^{(2)}$ M 2.O.	$\lambda_y^{(2)}$ F 2.O.	$q_x^{\lim}$ Males	$q_y^{\lim}$ Fem.
0	0.05000000	0.05000000	0.05806153	0.05519411	0.00000166	0.00000117
1	0.05000000	0.05000000	0.05433941	0.05423112	0.00000011	0.00000010
2	0.05000000	0.05000000	0.05143628	0.05407045	0.00000008	0.00000007
3	0.05000000	0.05000000	0.04983139	0.05394294	0.00000006	0.00000004
4	0.05000000	0.05000000	0.04990217	0.05371626	0.00000005	0.00000003
5	0.05000000	0.05000000	0.05133550	0.05283611	0.00000004	0.00000003
6	0.05000000	0.05000000	0.05297229	0.05002439	0.00000004	0.00000003
7	0.05000000	0.04888164	0.05357259	0.04588164	0.00000004	0.00000003
8	0.05000000	0.04449729	0.05252179	0.04149729	0.00000003	0.00000007
9	0.05000000	0.04128836	0.05035440	0.03828836	0.00000003	0.00000011
10	0.05000000	0.03928645	0.04856713	0.03628645	0.00000004	0.00000016
11	0.05000000	0.03736129	0.04716746	0.03436129	0.00000004	0.00000023
12	0.04854485	0.03561338	0.04554485	0.03261338	0.00000004	0.00000033
13	0.04710818	0.03446111	0.04410818	0.03146111	0.00000007	0.00000046
14	0.04547210	0.03377116	0.04247210	0.03077116	0.00000014	0.00000066
15	0.04328238	0.03316061	0.04028238	0.03016061	0.00000033	0.00000091
16	0.04063392	0.03243689	0.03763392	0.02943689	0.00000076	0.00000122
17	0.03732280	0.03124157	0.03432280	0.02824157	0.00000176	0.00000177
18	0.03419043	0.03117829	0.03119043	0.02686651	0.00000353	0.00000200
19	0.03181457	0.03111501	0.02881457	0.02581114	0.00000552	0.00000204
20	0.03020658	0.03105172	0.02720658	0.02486942	0.00000714	0.00000197
21	0.03015694	0.03098844	0.02594333	0.02455979	0.00000718	0.00000190
22	0.03010730	0.03092516	0.02488497	0.02464996	0.00000718	0.00000185
23	0.03005766	0.03086188	0.02383356	0.02555904	0.00000717	0.00000178
24	0.03000802	0.03079860	0.02297395	0.02665116	0.00000718	0.00000171
25	0.02995838	0.03073531	0.02278681	0.02728571	0.00000721	0.00000173
26	0.02990874	0.03067203	0.02328835	0.02742869	0.00000719	0.00000181
27	0.02985910	0.03060875	0.02424843	0.02803906	0.00000707	0.00000190
28	0.02980946	0.03054547	0.02540725	0.02853690	0.00000688	0.00000198
29	0.02975982	0.03048219	0.02658880	0.02887635	0.00000669	0.00000206
30	0.02971018	0.03041891	0.02739609	0.02923234	0.00000661	0.00000217
31	0.02966054	0.03035562	0.02785130	0.02930107	0.00000675	0.00000240
32	0.02961090	0.03029234	0.02783756	0.02899806	0.00000714	0.00000272
33	0.02956126	0.03022906	0.02758509	0.02857284	0.00000763	0.00000313
34	0.02951162	0.03016578	0.02704138	0.02816180	0.00000817	0.00000350
35	0.02946198	0.03010250	0.02643804	0.02748281	0.00000873	0.00000381
36	0.02941234	0.03003921	0.02602045	0.02656278	0.00000944	0.00000418
37	0.02936270	0.02997593	0.02566795	0.02550054	0.00001045	0.00000471
38	0.02931306	0.02991265	0.02529194	0.02454324	0.00001176	0.00000540
39	0.02926342	0.02984937	0.02484171	0.02373150	0.00001329	0.00000619
40	0.02921378	0.02978609	0.02452521	0.02282011	0.00001496	0.00000706
41	0.02916414	0.02972281	0.02452076	0.02166987	0.00001640	0.00000786
42	0.02911450	0.02965952	0.02471814	0.02041719	0.00001781	0.00000866
43	0.02906486	0.02959624	0.02494290	0.01941327	0.00001927	0.00000949
44	0.02901522	0.02953296	0.02476846	0.01884703	0.00002087	0.00001034
45	0.02896558	0.02946968	0.02432885	0.01852947	0.00002268	0.00001127
46	0.02891594	0.02940640	0.02391112	0.01846621	0.00002472	0.00001237
47	0.02886630	0.02934311	0.02349376	0.01849895	0.00002697	0.00001363

Age	Initial trend, 1st Order		Initial trend, 2nd Order		Limiting probabilities	
	λ_x Males	λ_y Fem.	$\lambda_x^{(2)}$ M 2.O.	$\lambda_y^{(2)}$ F 2.O.	$q_x^{\lim}$ Males	$q_y^{\lim}$ Fem.
48	0.02881666	0.02927983	0.02308607	0.01864868	0.00002947	0.00001495
49	0.02876702	0.02921655	0.02258214	0.01900263	0.00003228	0.00001632
50	0.02871738	0.02915327	0.02188581	0.01911961	0.00003539	0.00001777
51	0.02866774	0.02908999	0.02118548	0.01926948	0.00003870	0.00001927
52	0.02861810	0.02902671	0.02072265	0.01944860	0.00004207	0.00002077
53	0.02856846	0.02896342	0.02052170	0.01956881	0.00004538	0.00002221
54	0.02851882	0.02890014	0.02062742	0.01990743	0.00004857	0.00002359
55	0.02846918	0.02883686	0.02088954	0.02053770	0.00005165	0.00002495
56	0.02841954	0.02877358	0.02123102	0.02128907	0.00005466	0.00002630
57	0.02836990	0.02871030	0.02140989	0.02213062	0.00005765	0.00002762
58	0.02832026	0.02864701	0.02157279	0.02314993	0.00006067	0.00002894
59	0.02827062	0.02858373	0.02174188	0.02411437	0.00006384	0.00003037
60	0.02822098	0.02852045	0.02174142	0.02484238	0.00006734	0.00003205
61	0.02817134	0.02845717	0.02170426	0.02527291	0.00007351	0.00003485
62	0.02812170	0.02839389	0.02160020	0.02539389	0.00008085	0.00003821
63	0.02807205	0.02839284	0.02158052	0.02539284	0.00008959	0.00004187
64	0.02802241	0.02846732	0.02149253	0.02546732	0.00009984	0.00004571
65	0.02797277	0.02861996	0.02153146	0.02561996	0.00011158	0.00004961
66	0.02792313	0.02870307	0.02177625	0.02570307	0.00012484	0.00005477
67	0.02787349	0.02879579	0.02216805	0.02579579	0.00013973	0.00006066
68	0.02782385	0.02893162	0.02254376	0.02593162	0.00015644	0.00006689
69	0.02777421	0.02922081	0.02294471	0.02622081	0.00017518	0.00007215
70	0.02772457	0.02953219	0.02339252	0.02653219	0.00019617	0.00007777
71	0.02767493	0.02988619	0.02388009	0.02688619	0.00021976	0.00008353
72	0.02762529	0.03015902	0.02431679	0.02715902	0.00024638	0.00009118
73	0.02757565	0.03037460	0.02454015	0.02737460	0.00027663	0.00010077
74	0.02754015	0.03046444	0.02452601	0.02746444	0.00031051	0.00011399
75	0.02754015	0.03046444	0.02424996	0.02740030	0.00034745	0.00013115
76	0.02754015	0.03046444	0.02380594	0.02720161	0.00038982	0.00015121
77	0.02754015	0.03046444	0.02323639	0.02685175	0.00043856	0.00017463
78	0.02752601	0.03046444	0.02254476	0.02643356	0.00049585	0.00020190
79	0.02724996	0.03040030	0.02182283	0.02586915	0.00058581	0.00023603
80	0.02680594	0.03020161	0.02099652	0.02512303	0.00071258	0.00028203
81	0.02623639	0.02985175	0.02003335	0.02425875	0.00088645	0.00034527
82	0.02554476	0.02943356	0.01893564	0.02325261	0.00112700	0.00042743
83	0.02482283	0.02886915	0.01778166	0.02210306	0.00143375	0.00054162
84	0.02399652	0.02812303	0.01660786	0.02079642	0.00183977	0.00070637
85	0.02303335	0.02725875	0.01547357	0.01938759	0.00240326	0.00093869
86	0.02193564	0.02625261	0.01442225	0.01796214	0.00319846	0.00127570
87	0.02078166	0.02510306	0.01340182	0.01654835	0.00429208	0.00176266
88	0.01960786	0.02379642	0.01246408	0.01516514	0.00578076	0.00248169
89	0.01847357	0.02238759	0.01148995	0.01386080	0.00775377	0.00354459
90	0.01742225	0.02096214	0.01044269	0.01266659	0.01028397	0.00506686
91	0.01640182	0.01954835	0.00922911	0.01159572	0.01358998	0.00721127
92	0.01546408	0.01816514	0.00789115	0.01047178	0.01771129	0.01018732
93	0.01448995	0.01686080	0.00650226	0.00929733	0.02314931	0.01417105
94	0.01344269	0.01566659	0.00532700	0.00813900	0.03047981	0.01929495
95	0.01222911	0.01459572	0.00431235	0.00701414	0.04099885	0.02562873
96	0.01089115	0.01347178	0.00329769	0.00588928	0.05596862	0.03410885
97	0.00950226	0.01229733	0.00228303	0.00476442	0.07667193	0.04549671
98	0.00832700	0.01113900	0.00126838	0.00363956	0.10114232	0.06023426

Age	Initial trend, 1st Order		Initial trend, 2nd Order		Limiting probabilities	
	λ_x Males	λ_y Fem.	$\lambda_x^{(2)}$ M 2.O.	$\lambda_y^{(2)}$ F 2.O.	$q_x^{\lim}$ Males	$q_y^{\lim}$ Fem.
99	0.00731235	0.01001414	0.00025372	0.00251470	0.12959522	0.07897055
100	0.00636494	0.00895016	0.00000000	0.00138984	0.16371737	0.10211956
101	0.00554028	0.00799922	0.00000000	0.00026498	0.19730966	0.12634014
102	0.00482247	0.00714932	0.00000000	0.00000000	0.23312955	0.15327315
103	0.00419766	0.00638971	0.00000000	0.00000000	0.27070904	0.18271307
104	0.00365380	0.00571082	0.00000000	0.00000000	0.30959816	0.21441723
105	0.00318040	0.00510406	0.00000000	0.00000000	0.34938213	0.24811735
106	0.00276834	0.00456176	0.00000000	0.00000000	0.38969126	0.28352816
107	0.00240967	0.00407708	0.00000000	0.00000000	0.43020669	0.32036196
108	0.00209747	0.00364390	0.00000000	0.00000000	0.47066256	0.35833872
109	0.00182571	0.00325674	0.00000000	0.00000000	0.51084387	0.39719316
110	0.00158917	0.00291072	0.00000000	0.00000000	0.55058350	0.43668037
111	0.00138327	0.00260146	0.00000000	0.00000000	0.58975647	0.47657913
112	0.00120405	0.00232506	0.00000000	0.00000000	0.62827488	0.51669377
113	0.00104805	0.00207803	0.00000000	0.00000000	0.66801526	0.56121896
114	0.00091226	0.00185724	0.00000000	0.00000000	0.70245336	0.59987493
115	0.00079407	0.00165991	0.00000000	0.00000000	0.73491370	0.63713230
116	0.00069119	0.00148355	0.00000000	0.00000000	0.76527505	0.67271094
117	0.00060164	0.00132592	0.00000000	0.00000000	0.79345203	0.70637521
118	0.00052369	0.00118505	0.00000000	0.00000000	0.81939433	0.73793743
119	0.00045584	0.00105914	0.00000000	0.00000000	0.84308550	0.76725989
120	0.00039678	0.00094661	0.00000000	0.00000000	0.86454126	0.79425538

If values for ages above $x = 120$ years are required, they can easily be obtained from Equation (13). For all practical calculations, we recommend to introduce a maximum age of 120 or 121 years.

A.4 Values of the Long-Term Trend Reduction $G(t)$

Year t	$R(t)$	$G(t)$
2001	1.000	0.000
2002	1.000	1.000
2003	1.000	2.000
2004	0.999	2.999
2005	0.998	3.998
2006	0.998	4.996
2007	0.996	5.993
2008	0.995	6.989
2009	0.994	7.983
2010	0.992	8.976
2011	0.990	9.967
2012	0.988	10.956
2013	0.986	11.943
2014	0.983	12.928
2015	0.981	13.910
2016	0.978	14.889
2017	0.975	15.866
2018	0.972	16.839
2019	0.969	17.809
2020	0.965	18.776
2021	0.962	19.740
2022	0.958	20.699
2023	0.954	21.655
2024	0.950	22.607
2025	0.946	23.554
2026	0.941	24.498
2027	0.937	25.437
2028	0.932	26.371
2029	0.927	27.301
2030	0.922	28.226
2031	0.917	29.146
2032	0.912	30.061
2033	0.907	30.970
2034	0.902	31.875
2035	0.896	32.774
2036	0.891	33.667
2037	0.885	34.556
2038	0.880	35.438
2039	0.874	36.315
2040	0.868	37.186
2041	0.862	38.051
2042	0.856	38.910
2043	0.850	39.763
2044	0.844	40.610
2045	0.838	41.451
2046	0.832	42.285
2047	0.825	43.114
2048	0.819	43.936
2049	0.813	44.752
2050	0.806	45.562

Year t	$R(t)$	$G(t)$
2051	0.800	46.365
2052	0.794	47.162
2053	0.787	47.952
2054	0.781	48.736
2055	0.774	49.513
2056	0.768	50.284
2057	0.761	51.049
2058	0.755	51.807
2059	0.748	52.558
2060	0.742	53.303
2061	0.735	54.042
2062	0.729	54.774
2063	0.722	55.500
2064	0.716	56.219
2065	0.709	56.931
2066	0.703	57.638
2067	0.697	58.337
2068	0.690	59.031
2069	0.684	59.718
2070	0.677	60.398
2071	0.671	61.073
2072	0.665	61.741
2073	0.659	62.402
2074	0.652	63.058
2075	0.646	63.707
2076	0.640	64.350
2077	0.634	64.987
2078	0.628	65.618
2079	0.622	66.243
2080	0.616	66.861
2081	0.610	67.474
2082	0.604	68.081
2083	0.598	68.682
2084	0.592	69.277
2085	0.586	69.866
2086	0.581	70.449
2087	0.575	71.027
2088	0.569	71.599
2089	0.564	72.165
2090	0.558	72.726
2091	0.552	73.282
2092	0.547	73.831
2093	0.542	74.376
2094	0.536	74.914
2095	0.531	75.448
2096	0.526	75.976
2097	0.520	76.499
2098	0.515	77.017
2099	0.510	77.530
2100	0.505	78.037

Year t	$R(t)$	$G(t)$
2101	0.500	78.540
2102	0.495	79.037
2103	0.490	79.530
2104	0.485	80.018
2105	0.480	80.500
2106	0.476	80.978
2107	0.471	81.452
2108	0.466	81.920
2109	0.462	82.384
2110	0.457	82.843
2111	0.452	83.298
2112	0.448	83.748
2113	0.444	84.194
2114	0.439	84.636
2115	0.435	85.073
2116	0.431	85.505
2117	0.426	85.934
2118	0.422	86.358
2119	0.418	86.778
2120	0.414	87.194
2121	0.410	87.606
2122	0.406	88.014
2123	0.402	88.417
2124	0.398	88.817
2125	0.394	89.213
2126	0.390	89.606
2127	0.386	89.994
2128	0.383	90.378
2129	0.379	90.759
2130	0.375	91.137
2131	0.372	91.510
2132	0.368	91.880
2133	0.365	92.246
2134	0.361	92.609
2135	0.358	92.969
2136	0.354	93.325
2137	0.351	93.677
2138	0.348	94.027
2139	0.344	94.373
2140	0.341	94.715
2141	0.338	95.055
2142	0.335	95.391
2143	0.332	95.724
2144	0.328	96.054
2145	0.325	96.381
2146	0.322	96.705
2147	0.319	97.026
2148	0.316	97.343
2149	0.313	97.658
2150	0.311	97.970

A.5 The Approximated Table Using Age Shift

Formula for the death probability using age shift relative to the generation $\tau_0 = 1965$ of an annuitant aged x and born in year $\tau = 1905, 1906, \ldots, 2020$:

$$q_x(\tau) = q^{\mathrm{AS,base}}_{x+\Delta(\tau)}(\tau_0). \tag{17}$$

The base table $q^{\mathrm{AS,base}}_x(1965)$ for the generation 1965 and the age shifts $\Delta(\tau)$ relative to that generation are tabulated in the following sections.

A.6 Base Table $q^{\mathrm{AS,base}}_x(1965)$ for the Age Shift

	Individual contracts		Group contracts	
Age	$q^{\mathrm{AS,base}}_x$ Males	$q^{\mathrm{AS,base}}_y$ Fem.	$q^{\mathrm{AS,base}}_x$ M Gr.	$q^{\mathrm{AS,base}}_y$ F Gr.
0	0.000230	0.000150	0.000230	0.000150
1	0.000230	0.000150	0.000230	0.000150
2	0.000230	0.000150	0.000230	0.000150
3	0.000230	0.000150	0.000230	0.000150
4	0.000230	0.000150	0.000230	0.000150
5	0.000230	0.000150	0.000230	0.000150
6	0.000230	0.000150	0.000230	0.000150
7	0.000230	0.000150	0.000230	0.000150
8	0.000230	0.000150	0.000230	0.000150
9	0.000230	0.000150	0.000230	0.000150
10	0.000230	0.000150	0.000230	0.000150
11	0.000230	0.000150	0.000230	0.000150
12	0.000230	0.000150	0.000230	0.000150
13	0.000290	0.000150	0.000290	0.000150
14	0.000290	0.000150	0.000290	0.000150
15	0.000529	0.000227	0.000529	0.000227
16	0.000726	0.000227	0.000726	0.000227
17	0.000726	0.000227	0.000726	0.000227
18	0.000726	0.000227	0.000726	0.000227
19	0.000726	0.000227	0.000726	0.000227
20	0.000726	0.000227	0.000726	0.000227
21	0.000726	0.000227	0.000726	0.000227
22	0.000726	0.000227	0.000726	0.000227
23	0.000726	0.000227	0.000726	0.000227
24	0.000726	0.000227	0.000726	0.000227
25	0.000726	0.000227	0.000726	0.000227
26	0.000726	0.000278	0.000726	0.000278
27	0.000726	0.000278	0.000726	0.000278
28	0.000726	0.000298	0.000726	0.000298
29	0.000726	0.000298	0.000726	0.000298
30	0.000726	0.000298	0.000726	0.000298
31	0.000726	0.000298	0.000726	0.000298
32	0.000726	0.000298	0.000726	0.000298
33	0.000741	0.000298	0.000741	0.000298
34	0.000741	0.000377	0.000741	0.000377
35	0.000818	0.000377	0.000818	0.000377
36	0.000958	0.000468	0.000958	0.000468

Age	Individual contracts		Group contracts	
	$q_x^{\text{AS,base}}$ Males	$q_y^{\text{AS,base}}$ Fem.	$q_x^{\text{AS,base}}$ M Gr.	$q_y^{\text{AS,base}}$ F Gr.
37	0.001022	0.000507	0.001022	0.000507
38	0.001109	0.000559	0.001109	0.000559
39	0.001207	0.000615	0.001207	0.000615
40	0.001310	0.000675	0.001310	0.000675
41	0.001384	0.000722	0.001392	0.000726
42	0.001449	0.000765	0.001468	0.000773
43	0.001512	0.000806	0.001542	0.000819
44	0.001579	0.000845	0.001622	0.000863
45	0.001655	0.000886	0.001712	0.000911
46	0.001740	0.000936	0.001813	0.000968
47	0.001831	0.000992	0.001923	0.001032
48	0.001931	0.001048	0.002044	0.001097
49	0.002041	0.001101	0.002178	0.001160
50	0.002160	0.001155	0.002325	0.001225
51	0.002280	0.001206	0.002476	0.001289
52	0.002394	0.001252	0.002623	0.001348
53	0.002494	0.001290	0.002759	0.001399
54	0.002578	0.001321	0.002881	0.001443
55	0.002649	0.001346	0.002990	0.001482
56	0.002709	0.001368	0.003090	0.001519
57	0.002761	0.001386	0.003185	0.001551
58	0.002809	0.001401	0.003278	0.001582
59	0.002858	0.001418	0.003376	0.001616
60	0.002916	0.001445	0.003488	0.001661
61	0.003079	0.001516	0.003682	0.001743
62	0.003277	0.001605	0.003916	0.001845
63	0.003514	0.001713	0.004195	0.001967
64	0.003791	0.001839	0.004519	0.002109
65	0.004102	0.001982	0.004882	0.002271
66	0.004444	0.002154	0.005278	0.002465
67	0.004818	0.002352	0.005708	0.002685
68	0.005227	0.002570	0.006174	0.002928
69	0.005672	0.002800	0.006679	0.003183
70	0.006157	0.003056	0.007224	0.003465
71	0.006687	0.003339	0.007817	0.003775
72	0.007271	0.003670	0.008465	0.004137
73	0.007919	0.004055	0.009180	0.004556
74	0.008639	0.004515	0.009969	0.005055
75	0.009438	0.005058	0.010839	0.005643
76	0.010340	0.005681	0.011816	0.006314
77	0.011361	0.006391	0.012916	0.007075
78	0.012526	0.007199	0.014165	0.007938
79	0.014000	0.008141	0.015747	0.008939
80	0.015811	0.009267	0.017685	0.010132
81	0.018016	0.010627	0.020039	0.011569
82	0.020714	0.012237	0.022909	0.013263
83	0.023780	0.014199	0.026149	0.015321
84	0.027252	0.016633	0.029793	0.017866
85	0.031358	0.019620	0.034082	0.020980
86	0.036280	0.023337	0.039202	0.024840
87	0.042142	0.027830	0.045271	0.029485

Age	Individual contracts		Group contracts	
	$q_x^{\text{AS,base}}$ Males	$q_y^{\text{AS,base}}$ Fem.	$q_x^{\text{AS,base}}$ M Gr.	$q_y^{\text{AS,base}}$ F Gr.
88	0.049121	0.033308	0.052461	0.035127
89	0.057379	0.040084	0.060925	0.042079
90	0.066996	0.048305	0.070727	0.050476
91	0.078333	0.058167	0.082223	0.060503
92	0.091274	0.069910	0.095264	0.072388
93	0.106417	0.083612	0.110446	0.086185
94	0.124227	0.099220	0.128217	0.101815
95	0.145858	0.116560	0.149720	0.119079
96	0.171913	0.136682	0.175515	0.139025
97	0.202689	0.160088	0.205838	0.162128
98	0.235659	0.186745	0.238072	0.188316
99	0.270953	0.216829	0.272324	0.217733
100	0.309692	0.250257	0.309692	0.250257
101	0.342308	0.279901	0.342308	0.279901
102	0.375319	0.310493	0.375319	0.310493
103	0.408553	0.341867	0.408553	0.341867
104	0.441870	0.373882	0.441870	0.373882
105	0.475160	0.406413	0.475160	0.406413
106	0.508332	0.439339	0.508332	0.439339
107	0.541315	0.472556	0.541315	0.472556
108	0.574054	0.505968	0.574054	0.505968
109	0.606510	0.539492	0.606510	0.539492
110	0.638654	0.573056	0.638654	0.573056
111	0.670471	0.606598	0.670471	0.606598
112	0.701950	0.640066	0.701950	0.640066
113	0.735219	0.678694	0.735219	0.678694
114	0.763144	0.710115	0.763144	0.710115
115	0.789494	0.740060	0.789494	0.740060
116	0.814170	0.768354	0.814170	0.768354
117	0.837095	0.794849	0.837095	0.794849
118	0.858218	0.819435	0.858218	0.819435
119	0.877513	0.842034	0.877513	0.842034
120	0.894982	0.862610	0.894982	0.862610
121	1	1	1	1

A.7 Age Shifts $\Delta(\tau)$ Applied to the Base Table 1965

The age shift for birth years $\tau < 1920$ is monotonized to ensure non-increasing actuarial values as discussed in Section 4.10.

	Individual contracts				Group contracts			
Birth year τ	Males	rounded	Females	rounded	Males	rounded	Females	rounded
1905	4.28	4	4.58	5	4.47	4	4.73	5
1906	4.28	4	4.58	5	4.47	4	4.73	5
1907	4.28	4	4.58	5	4.47	4	4.73	5
1908	4.28	4	4.58	5	4.47	4	4.73	5
1909	4.28	4	4.58	5	4.47	4	4.73	5
1910	4.28	4	4.58	5	4.47	4	4.73	5
1911	4.28	4	4.58	5	4.47	4	4.73	5
1912	4.28	4	4.58	5	4.47	4	4.73	5
1913	4.28	4	4.58	5	4.47	4	4.73	5
1914	4.28	4	4.58	5	4.47	4	4.73	5
1915	4.28	4	4.58	5	4.47	4	4.73	5
1916	4.28	4	4.58	5	4.47	4	4.73	5
1917	4.28	4	4.58	5	4.47	4	4.73	5
1918	4.28	4	4.58	5	4.47	4	4.73	5
1919	4.28	4	4.58	5	4.47	4	4.73	5
1920	4.28	4	4.57	5	4.47	4	4.72	5
1921	4.27	4	4.52	5	4.46	4	4.67	5
1922	4.26	4	4.47	4	4.45	4	4.62	5
1923	4.24	4	4.42	4	4.44	4	4.57	5
1924	4.22	4	4.37	4	4.42	4	4.51	5
1925	4.19	4	4.31	4	4.39	4	4.45	4
1926	4.15	4	4.25	4	4.36	4	4.39	4
1927	4.11	4	4.18	4	4.31	4	4.32	4
1928	4.06	4	4.11	4	4.27	4	4.25	4
1929	4.01	4	4.03	4	4.22	4	4.18	4
1930	3.95	4	3.96	4	4.16	4	4.10	4
1931	3.89	4	3.87	4	4.10	4	4.02	4
1932	3.83	4	3.79	4	4.04	4	3.93	4
1933	3.76	4	3.70	4	3.97	4	3.84	4
1934	3.69	4	3.61	4	3.90	4	3.75	4
1935	3.65	4	3.54	4	3.86	4	3.68	4
1936	3.60	4	3.46	3	3.81	4	3.60	4
1937	3.53	4	3.37	3	3.75	4	3.51	4
1938	3.46	3	3.28	3	3.68	4	3.42	3
1939	3.38	3	3.18	3	3.60	4	3.32	3
1940	3.29	3	3.08	3	3.51	4	3.21	3
1941	3.20	3	2.97	3	3.41	3	3.10	3
1942	3.10	3	2.86	3	3.30	3	2.99	3
1943	2.99	3	2.75	3	3.19	3	2.87	3
1944	2.88	3	2.63	3	3.08	3	2.75	3
1945	2.76	3	2.51	3	2.96	3	2.63	3
1946	2.61	3	2.37	2	2.80	3	2.49	2
1947	2.47	2	2.24	2	2.64	3	2.34	2
1948	2.32	2	2.10	2	2.49	2	2.20	2
1949	2.18	2	1.97	2	2.33	2	2.06	2
1950	2.03	2	1.83	2	2.18	2	1.92	2
1951	1.89	2	1.70	2	2.03	2	1.79	2
1952	1.75	2	1.57	2	1.88	2	1.65	2
1953	1.61	2	1.44	1	1.73	2	1.51	2
1954	1.47	1	1.32	1	1.58	2	1.38	1
1955	1.33	1	1.19	1	1.43	1	1.25	1
1956	1.19	1	1.06	1	1.28	1	1.12	1
1957	1.05	1	0.94	1	1.13	1	0.98	1
1958	0.91	1	0.81	1	0.98	1	0.85	1
1959	0.78	1	0.69	1	0.83	1	0.73	1
1960	0.64	1	0.57	1	0.69	1	0.60	1

Birth year τ	Individual contracts				Group contracts			
	Males	rounded	Females	rounded	Males	rounded	Females	rounded
1961	0.51	1	0.45	0	0.55	1	0.48	0
1962	0.38	0	0.34	0	0.41	0	0.35	0
1963	0.25	0	0.22	0	0.27	0	0.23	0
1964	0.12	0	0.11	0	0.13	0	0.12	0
1965	0.00	0	0.00	0	0.00	0	0.00	0
1966	−0.12	0	−0.11	0	−0.13	0	−0.12	0
1967	−0.24	0	−0.22	0	−0.26	0	−0.23	0
1968	−0.36	0	−0.32	0	−0.39	0	−0.34	0
1969	−0.48	0	−0.43	0	−0.52	−1	−0.45	0
1970	−0.60	−1	−0.53	−1	−0.64	−1	−0.55	−1
1971	−0.71	−1	−0.63	−1	−0.76	−1	−0.66	−1
1972	−0.82	−1	−0.73	−1	−0.88	−1	−0.76	−1
1973	−0.93	−1	−0.82	−1	−1.00	−1	−0.86	−1
1974	−1.04	−1	−0.92	−1	−1.11	−1	−0.96	−1
1975	−1.14	−1	−1.01	−1	−1.23	−1	−1.06	−1
1976	−1.25	−1	−1.11	−1	−1.34	−1	−1.16	−1
1977	−1.35	−1	−1.20	−1	−1.45	−1	−1.26	−1
1978	−1.45	−1	−1.29	−1	−1.56	−2	−1.35	−1
1979	−1.55	−2	−1.37	−1	−1.67	−2	−1.44	−1
1980	−1.65	−2	−1.46	−1	−1.77	−2	−1.53	−2
1981	−1.74	−2	−1.54	−2	−1.87	−2	−1.62	−2
1982	−1.84	−2	−1.63	−2	−1.98	−2	−1.71	−2
1983	−1.93	−2	−1.71	−2	−2.08	−2	−1.79	−2
1984	−2.02	−2	−1.79	−2	−2.17	−2	−1.88	−2
1985	−2.11	−2	−1.87	−2	−2.27	−2	−1.96	−2
1986	−2.20	−2	−1.95	−2	−2.37	−2	−2.04	−2
1987	−2.29	−2	−2.02	−2	−2.46	−2	−2.12	−2
1988	−2.37	−2	−2.10	−2	−2.55	−3	−2.20	−2
1989	−2.46	−2	−2.17	−2	−2.64	−3	−2.28	−2
1990	−2.54	−3	−2.25	−2	−2.73	−3	−2.35	−2
1991	−2.62	−3	−2.32	−2	−2.82	−3	−2.43	−2
1992	−2.70	−3	−2.39	−2	−2.90	−3	−2.50	−3
1993	−2.78	−3	−2.46	−2	−2.99	−3	−2.57	−3
1994	−2.86	−3	−2.52	−3	−3.07	−3	−2.65	−3
1995	−2.94	−3	−2.59	−3	−3.15	−3	−2.72	−3
1996	−3.01	−3	−2.66	−3	−3.23	−3	−2.79	−3
1997	−3.08	−3	−2.72	−3	−3.31	−3	−2.85	−3
1998	−3.16	−3	−2.78	−3	−3.39	−3	−2.92	−3
1999	−3.23	−3	−2.85	−3	−3.47	−3	−2.99	−3
2000	−3.30	−3	−2.91	−3	−3.54	−4	−3.05	−3
2001	−3.37	−3	−2.97	−3	−3.62	−4	−3.11	−3
2002	−3.43	−3	−3.03	−3	−3.69	−4	−3.18	−3
2003	−3.50	−4	−3.09	−3	−3.76	−4	−3.24	−3
2004	−3.57	−4	−3.14	−3	−3.83	−4	−3.30	−3
2005	−3.63	−4	−3.20	−3	−3.90	−4	−3.36	−3
2006	−3.69	−4	−3.26	−3	−3.97	−4	−3.41	−3
2007	−3.76	−4	−3.31	−3	−4.04	−4	−3.47	−3
2008	−3.82	−4	−3.37	−3	−4.10	−4	−3.53	−4
2009	−3.88	−4	−3.42	−3	−4.17	−4	−3.58	−4
2010	−3.94	−4	−3.47	−3	−4.23	−4	−3.64	−4
2011	−4.00	−4	−3.52	−4	−4.30	−4	−3.69	−4
2012	−4.05	−4	−3.57	−4	−4.36	−4	−3.75	−4
2013	−4.11	−4	−3.62	−4	−4.42	−4	−3.80	−4
2014	−4.17	−4	−3.67	−4	−4.48	−4	−3.85	−4
2015	−4.22	−4	−3.72	−4	−4.54	−5	−3.90	−4
2016	−4.28	−4	−3.77	−4	−4.60	−5	−3.95	−4
2017	−4.33	−4	−3.82	−4	−4.65	−5	−4.00	−4
2018	−4.38	−4	−3.86	−4	−4.71	−5	−4.05	−4
2019	−4.43	−4	−3.91	−4	−4.77	−5	−4.09	−4
2020	−4.48	−4	−3.95	−4	−4.82	−5	−4.14	−4

A.8 Net Single Premiums Calculated from the Base Table 1965 for the Age Shift

Net single premiums $\ddot{a}_x^{\text{AS,base}}(1965)$ of life annuity-dues of 1 calculated with yearly effective interest rate $r = 2.25\%$ from the base tables for the generation 1965 of the age shift. As only the age range from 50 to 90 years was used for the fit of the age shift, results for ages outside this range might differ considerably from those of the exact table. Additionally, since the base table was monotonized, the values for ages $x \leq 20$ years given in this table are not the exact values of the generation $\tau = 1965$.

For these reasons, this table is only given for convenience. For all relevant calculations we strongly recommend the use of the exact table.

	Individual contracts		Group contracts	
Age	$\ddot{a}_x^{\text{AS,base}}$ M	$\ddot{a}_y^{\text{AS,base}}$ F	$\ddot{a}_x^{\text{AS,base}}$ M Gr	$\ddot{a}_y^{\text{AS,base}}$ F Gr.
0	33.5116	34.0868	33.4303	34.0445
1	33.4134	34.0017	33.3298	33.9583
2	33.3124	33.9144	33.2265	33.8698
3	33.2086	33.8246	33.1204	33.7787
4	33.1019	33.7323	33.0112	33.6852
5	32.9923	33.6375	32.8991	33.5891
6	32.8797	33.5400	32.7839	33.4903
7	32.7639	33.4399	32.6654	33.3887
8	32.6449	33.3369	32.5437	33.2844
9	32.5226	33.2312	32.4186	33.1772
10	32.3969	33.1225	32.2900	33.0670
11	32.2677	33.0108	32.1579	32.9538
12	32.1349	32.8960	32.0221	32.8374
13	31.9985	32.7781	31.8825	32.7179
14	31.8602	32.6568	31.7409	32.5950
15	31.7180	32.5323	31.5955	32.4687
16	31.5795	32.4068	31.4535	32.3414
17	31.4432	32.2778	31.3137	32.2106
18	31.3032	32.1452	31.1700	32.0762
19	31.1591	32.0090	31.0222	31.9381
20	31.0111	31.8690	30.8702	31.7961
21	30.8588	31.7251	30.7140	31.6502
22	30.7022	31.5772	30.5533	31.5002
23	30.5412	31.4252	30.3881	31.3461
24	30.3756	31.2690	30.2182	31.1877
25	30.2054	31.1085	30.0435	31.0249
26	30.0304	30.9435	29.8639	30.8576
27	29.8504	30.7755	29.6792	30.6872
28	29.6653	30.6029	29.4893	30.5121
29	29.4750	30.4260	29.2941	30.3327
30	29.2794	30.2442	29.0933	30.1484
31	29.0782	30.0574	28.8869	29.9589
32	28.8713	29.8654	28.6746	29.7641
33	28.6586	29.6680	28.4563	29.5639
34	28.4403	29.4652	28.2323	29.3582
35	28.2158	29.2590	28.0019	29.1490
36	27.9871	29.0471	27.7672	28.9341
37	27.7558	28.8319	27.5296	28.7157
38	27.5198	28.6118	27.2871	28.4923
39	27.2793	28.3869	27.0400	28.2641
40	27.0346	28.1574	26.7884	28.0312
41	26.7857	27.9231	26.5323	27.7933

Age	Individual contracts		Group contracts	
	$\ddot{a}_x^{\mathrm{AS,base}}$ M	$\ddot{a}_y^{\mathrm{AS,base}}$ F	$\ddot{a}_x^{\mathrm{AS,base}}$ M Gr	$\ddot{a}_y^{\mathrm{AS,base}}$ F Gr.
42	26.5315	27.6834	26.2711	27.5501
43	26.2717	27.4382	26.0042	27.3013
44	26.0059	27.1872	25.7315	27.0467
45	25.7342	26.9301	25.4528	26.7862
46	25.4566	26.6668	25.1684	26.5194
47	25.1729	26.3973	24.8781	26.2466
48	24.8832	26.1217	24.5820	25.9677
49	24.5875	25.8396	24.2802	25.6825
50	24.2857	25.5508	23.9726	25.3907
51	23.9779	25.2551	23.6594	25.0922
52	23.6637	24.9522	23.3403	24.7867
53	23.3429	24.6418	23.0150	24.4738
54	23.0147	24.3233	22.6830	24.1531
55	22.6786	23.9964	22.3436	23.8242
56	22.3339	23.6606	21.9964	23.4867
57	21.9801	23.3157	21.6406	23.1402
58	21.6167	22.9612	21.2760	22.7844
59	21.2434	22.5968	20.9021	22.4189
60	20.8597	22.2222	20.5187	22.0436
61	20.4655	21.8374	20.1257	21.6583
62	20.0626	21.4429	19.7243	21.2634
63	19.6512	21.0389	19.3148	20.8592
64	19.2317	20.6253	18.8978	20.4455
65	18.8043	20.2021	18.4734	20.0225
66	18.3693	19.7693	18.0420	19.5901
67	17.9266	19.3271	17.6036	19.1485
68	17.4763	18.8755	17.1581	18.6978
69	17.0184	18.4144	16.7056	18.2379
70	16.5527	17.9436	16.2460	17.7685
71	16.0794	17.4629	15.7793	17.2895
72	15.5984	16.9723	15.3054	16.8009
73	15.1098	16.4720	14.8242	16.3029
74	14.6135	15.9622	14.3360	15.7957
75	14.1098	15.4434	13.8407	15.2798
76	13.5986	14.9160	13.3384	14.7558
77	13.0803	14.3804	12.8293	14.2239
78	12.5552	13.8368	12.3137	13.6843
79	12.0236	13.2854	11.7918	13.1374
80	11.4875	12.7269	11.2660	12.5837
81	10.9491	12.1621	10.7382	12.0241
82	10.4102	11.5922	10.2107	11.4598
83	9.8735	11.0184	9.6858	10.8919
84	9.3396	10.4421	9.1643	10.3221
85	8.8090	9.8659	8.6465	9.7527
86	8.2835	9.2920	8.1340	9.1861
87	7.7655	8.7236	7.6292	8.6255
88	7.2574	8.1632	7.1345	8.0733
89	6.7617	7.6138	6.6522	7.5324
90	6.2805	7.0794	6.1844	7.0068
91	5.8153	6.5637	5.7324	6.5001
92	5.3682	6.0697	5.2982	6.0153
93	4.9392	5.6007	4.8814	5.5554
94	4.5295	5.1585	4.4833	5.1221
95	4.1410	4.7436	4.1055	4.7156
96	3.7785	4.3540	3.7528	4.3338
97	3.4476	3.9919	3.4306	3.9787
98	3.1542	3.6601	3.1448	3.6528

Age	Individual contracts		Group contracts	
	$\ddot{a}_x^{\text{AS,base}}$ M	$\ddot{a}_y^{\text{AS,base}}$ F	$\ddot{a}_x^{\text{AS,base}}$ M Gr	$\ddot{a}_y^{\text{AS,base}}$ F Gr.
99	2.8959	3.3608	2.8923	3.3581
100	2.6720	3.0974	2.6720	3.0974
101	2.4888	2.8744	2.4888	2.8744
102	2.3259	2.6745	2.3259	2.6745
103	2.1808	2.4954	2.1808	2.4954
104	2.0514	2.3346	2.0514	2.3346
105	1.9357	2.1902	1.9357	2.1902
106	1.8318	2.0603	1.8318	2.0603
107	1.7383	1.9431	1.7383	1.9431
108	1.6540	1.8373	1.6540	1.8373
109	1.5775	1.7414	1.5775	1.7414
110	1.5080	1.6543	1.5080	1.6543
111	1.4446	1.5746	1.4446	1.5746
112	1.3863	1.5008	1.3863	1.5008
113	1.3319	1.4295	1.3319	1.4295
114	1.2879	1.3735	1.2879	1.3735
115	1.2487	1.3238	1.2487	1.3238
116	1.2140	1.2797	1.2140	1.2797
117	1.1833	1.2409	1.1833	1.2409
118	1.1561	1.2064	1.1561	1.2064
119	1.1314	1.1743	1.1314	1.1743
120	1.1022	1.1337	1.1022	1.1337
121	1.0000	1.0000	1.0000	1.0000

B Glossary

Accumulation phase Aufschubzeit, in der Prämien bezahlt werden

Age shift Altersverschiebung; die Person wird so behandelt als sei sie älter/jünger

Annuity valuation table Rententafel, Rentnersterblichkeitstafel

Annuitization Beginn der Rentenauszahlung

Annuitization phase Auszahlungsphase

Blue-collar worker Arbeiter

Compulsory social security Gesetzliche Sozialversicherung

Curtate future lifetime Gestutzte zukünftige Lebensdauer

Dynamic life table Generationentafel, Sterblichkeit ist nicht nur vom Alter, sondern auch vom Geburtsjahr abhängig

Force of mortality Sterbeintensität

Life table Sterbetafel

Longevity Langlebigkeit

Mortality Sterblichkeit

Remaining lifetime Verbleibende Restlebenszeit

Static life table Periodentafel, Sterblichkeit ist nur vom Alter abhängig; typischerweise z.B. Ergebnisse einer Volkszählung, welche die Werte zu einem festen Zeitpunkt angeben

White-collar worker Angestellter